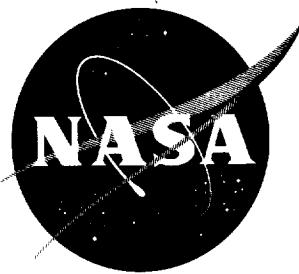


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NASA TN D-1319

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TECHNICAL NOTE

D-1319

THREE-DIMENSIONAL TRAJECTORY ANALYSIS FOR
ROUND-TRIP MISSIONS TO VENUS

By Charles L. Zola and Gerald Knip, Jr.

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON

August 1962

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SUMMARY

For orbit-to-orbit round-trip missions to Venus, the dependence of mission velocity-increment requirements on mission time, wait time, atmospheric braking, and synodic period of departure was investigated. The analysis is characterized by noncoplanar, elliptical planetary orbits; successive two-body approximations; and impulsive velocity increments. Round trips from 100 to 780 days and wait times from 40 to 510 days were studied for departure dates in 1979 and 1980.

The velocity-increment requirements for each round trip were based on three modes of operation: (1) atmospheric braking on arrival at Venus and Earth, (2) atmospheric braking on arrival at Earth, and (3) all-propulsive braking. All three velocity-increment summations were minimized for each mission time.

The dependence of velocity-increment requirements on mission time was such that a number of local minimum values of velocity-increment requirement occurred for mission times between 100 and 780 days. One of these minimums was for a mission time of about 460 days and a wait time of 40 days. A lower minimum occurred for a mission time of about 760 days and a wait time of 450 days. With atmospheric braking, the difference between these minimum values decreased. This trend may be significant in deciding between a long and a short round-trip mission to Venus.

For missions between 100 and 630 days, increasing the wait time increased the velocity-increment requirements. For missions between 630 and 780 days, however, increasing the wait time decreased the velocity-increment requirements.

The effect of synodic period of departure on velocity-increment requirements may be neglected for preliminary planning of round-trip missions to Venus.

INTRODUCTION

Of fundamental importance in the planning of a space mission as ambitious as an interplanetary round trip is a study of the trajectory requirements. These studies emphasize the interrelation of such parameters as mission time, wait time, atmospheric braking, synodic period of departure, and velocity-increment requirements.

Most previous round-trip interplanetary trajectory studies (e.g., refs. 1 to 3) have been based on a circular coplanar solar system. Because planetary orbital eccentricity and inclination are neglected in these studies, all values of trajectory variables are approximate. Reference 4 includes three-dimensional (characterized by mutually inclined, elliptical planetary orbits) effects for some arbitrary "nonstop" (zero wait time) round trips to Mars.

Three-dimensional trajectory data that will be useful in planning round-trip missions to Venus are presented herein. These data were calculated with the procedure of reference 5. Differences among data obtained with this procedure as compared with n-body calculations were small. For the missions investigated in reference 5, velocity-increment differences of less than 2 percent were obtained.

Round-trip mission times from 100 to 780 days and wait times from 40 to 510 days were studied for departure dates in 1979 and 1980. All trips started and ended in circular orbits at 1.1 planet radii. Total velocity-increment data for round trips employing three operational modes are presented: (1) atmospheric braking on arrival at both Venus and Earth, (2) atmospheric braking on arrival at Earth, and (3) all propulsive braking. A comparison of circular coplanar data and three-dimensional data (characterized by mutually inclined, elliptical planetary orbits) is presented.

ANALYSIS

The calculation procedure described in reference 5 was used in determining the required trajectory for each of the round trips investigated. This procedure is based on a three-dimensional model of the solar system that has mutually inclined, elliptical planetary orbits, successive two-body approximations, and impulsive velocity increments. A typical trajectory composed of three conic sections (escape hyperbola, heliocentric conic, and encounter hyperbola) starts and ends at the perigees of the respective hyperbolas.

For any round-trip mission, items such as communication distance, radiation shielding, sustenance weight, and reentry velocity all influence the choice of an optimum trip. Thus, velocity increment is only one factor, albeit a major one since the propellant is the heaviest constituent of the vehicle. The trajectory of prime interest, therefore,

is that one requiring the minimum velocity increment for any given mission and wait time. The four velocity increments required for the orbit-to-orbit round-trip mission are:

- (1) To depart a circular parking orbit at 1.1 Earth radii ΔV_1
- (2) To establish a circular parking orbit at 1.1 Venus radii for use during a given wait time ΔV_2
- (3) To depart the circular parking orbit at Venus ΔV_3
- (4) To establish a circular parking orbit at 1.1 Earth radii ΔV_4

These velocity increments are all assumed to be applied impulsively.

To determine the minimum velocity increment for a given value of mission time and wait time, a range of Earth-to-Venus trip time was investigated for a range of departure date. The results of reference 1 were used in selecting these initial ranges. After the values of $\Delta V_1 + \Delta V_2$ were obtained from curves of the form shown in figure 1(a), the required Venus-to-Earth trip times and the return departure dates were calculated from the following expressions:

$$T_{\oplus\ominus} = T_m - T_{\oplus\ominus} - T_w \quad (1)$$

$$D_{\oplus} = D_{\oplus} + T_{\oplus\ominus} + T_w \quad (2)$$

(All symbols are defined in the appendix.) Values of $\Delta V_3 + \Delta V_4$ were then obtained from curves of the form shown in figure 1(b). Combining the data of figures 1(a) and (b) gave the total velocity increment required for the mission as a function of departure date at Earth. These data are presented in curves of the form shown in figure 1(c). From the envelope curve of figure 1(c), a minimum value of the total velocity in-

crement $\sum_{i=1}^4 \Delta V_i$ required for the given mission time and wait time can be obtained.

One possible means of reducing the propulsion requirements is to employ atmospheric braking (ref. 6) either on arrival at Earth or on arrival at both Venus and Earth. The previous minimizing procedure was

therefore again applied to minimize $\sum_{i=1}^3 \Delta V_i$ for the case of

atmospheric braking on arrival at Earth and $\Delta V_1 + \Delta V_3$ for the case of atmospheric braking on arrival at Venus and Earth. Although ΔV_4 and $\Delta V_2 + \Delta V_4$ are omitted from these velocity-increment summations, small

velocity increments may still be required for orbit-to-orbit type missions to (1) establish the orbit when the vehicle glides up out of the atmosphere after obtaining approximately circular velocity, and (2) to initiate reentry for landing on the planet's surface.

RESULTS AND DISCUSSION

Ranges of round-trip mission time, wait time, and departure date were investigated for round trips between Earth and Venus. For each mission time and wait time, the minimum total velocity increments for three types of round trip were obtained. The three types are characterized by all propulsive braking, atmospheric braking at Earth, and atmospheric braking at both Venus and Earth.

Round Trips for Wait Time of 40 Days

For a wait time of 40 days, trajectory calculations were made for a range of Earth-to-Venus trip time of 20 to 280 days, a range of mission time of 100 to 700 days, and a range of departure date from Earth in 1979 and 1980. In addition to the total velocity increments for the three types of round trip, the individual velocity increments are presented. The effect of synodic period of departure on the round trip velocity-increment requirements is presented for three mission times.

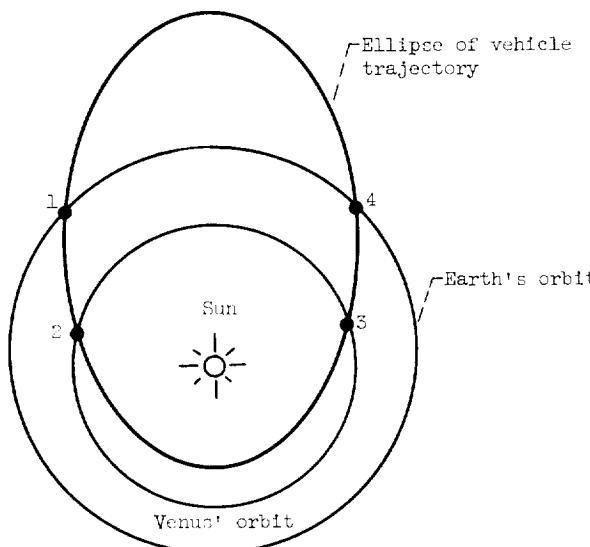
Velocity-increment requirements. - Figures 2 and 3 indicate individual velocity increments (ΔV_1 , ΔV_2 , ΔV_3 , and ΔV_4) for ranges of Earth-to-Venus and Venus-to-Earth trip times. In figure 2, a velocity-increment parameter is plotted against the date of departure from Earth (1980) for a range of Earth-to-Venus trip time. Figures 2(a) and (b) indicate values of ΔV_1 and ΔV_2 , respectively. Similarly in figure 3, a velocity-increment parameter is plotted against the date of departure from Venus in 1980-81 for a range of Venus-to-Earth trip time. The time of day for all departure dates is zero hours universal time (mean solar time of the meridian of Greenwich, reckoned from midnight).

Regions of high velocity increment occur for a trip requiring a heliocentric travel angle θ near 180° (ref. 5). As the travel angle approaches 180° , the inclination of the transfer plane increases rapidly. As the transfer-plane inclination increases, the velocity-increment requirements also increase. When the angle between Earth and Venus measured in the plane of the ecliptic is 180° , a maximum value of velocity increment occurs because an inclination of 90° is required of the transfer plane. The heliocentric travel angle in this case would be $180^\circ \pm \lambda_\oplus$ (latitude of Venus with respect to the ecliptic plane). This situation occurs on a 140-day trip to Venus (fig. 2(a)). The curve on the left is for a travel angle less than 180° , while the one on the right is for a travel angle greater than 180° . A means of bypassing

this difficulty of a single-plane transfer is to use a two-plane transfer. This type of transfer will not produce a minimum total-velocity increment for the particular mission time (ref. 5) but may be of use if launch is required during this period.

Total velocity-increment requirements are presented in figures 4 to 13 for mission times of 100, 150, 240, 300, 400, 460, 500, 580, 660, and 700 days. The total velocity increments for the three types of round trip are indicated in parts (a), (b), and (c) of the various figures. In each figure, a velocity-increment parameter is plotted against departure date from Earth for several values of Earth-to-Venus trip time. The time of day for all departure dates is zero hours universal time. For a given value of mission time and a specific type of round trip, the minimum value of total velocity increment can be determined from part (a), (b), or (c). The corresponding value of departure date, wait time, and Earth-to-Venus trip time permits the individual velocity increments to be read from figures 2 and 3. For example, figure 4(a) indicates the velocity-increment requirements for a 100-day mission employing atmospheric braking at Venus and Earth. A minimum mission velocity increment of 23.5 miles per second (1 mile = 0.8689762 int. naut. mile) results for a departure date of April 1, 1980 and an Earth-to-Venus trip time of 35 days. With this departure date, wait time, and Earth-to-Venus trip time, ΔV_1 (12 miles/sec) can be obtained from figure 2(a). With the aid of equations (1) and (2), ΔV_3 (11.5 miles/sec) can be obtained from figure 3(a).

From parts (a), (b), and (c) of figures 4 to 13, the minimum velocity increment for each mission time (100 to 700 days) can be obtained for each of the three types of round trip. These minimum values of total velocity increment are shown by the three envelope curves of figure 14. As discussed in reference 1, Earth-to-Venus and Venus-to-Earth trajectories (direct, aphelion, perihelion, and indirect (see sketch) may be combined in a number of ways. For each mission time, the total



Route	Earth to Venus	Venus to Earth
Direct	1 - 2	3 - 4
Perihelion	1 - 2 - 3	2 - 3 - 4
Aphelion	4 - 1 - 2	3 - 4 - 1
Indirect	4 - 1 - 2 - 3	2 - 3 - 4 - 1

velocity increment shown is for that combination which resulted in the minimum total-velocity increment. A cusp in the curves of minimum total velocity increment (mission time, approx. 500 days) results because of changing from one combination to another (ref. 1). Figure 14 points out that the total velocity increment does not decrease continuously with mission time. For a wait time of 40 days, a round-trip mission of about 460 days in 1980 resulted in the lowest values of velocity increment for the three types of round trip considered.

Synodic period effect. - Because the planetary orbits are eccentric, the synodic period of departure can have a sizeable effect on the velocity-increment requirements. For example, approximately 1-mile-per-second additional increment in velocity is required for a 150-day trip to Mars in 1962 compared with 1971 (ref. 5). The differences for Venus trips are expected to be less because the eccentricity of Venus' orbit (0.0068) is much less than the eccentricity of Mars' orbit (0.0934). Venus has a variation in orbital radius of about 1 million miles and a variation in velocity of 0.3 mile per second between perihelion and aphelion. Mars, however, has a variation in radius and velocity of 26 million miles and 3 miles per second, respectively.

In figure 14 are plotted the minimum total velocity increments for mission times of 240, 400, and 460 days with departure dates in 1965. Trips to Venus should take place during a span of several months near the inferior conjunction date (date on which the heliocentric longitudes of Earth and Venus are equal). For Venus, this date occurs about every 19 months or 1.6 years (synodic period). Earth is near its perihelion at the Venus opposition date in 1966 (January) and near its aphelion at the Venus opposition date in 1980 (June). The velocity-increment data for these synodic periods differ only slightly; the largest difference occurs for the 240-day mission with all propulsive braking. This difference amounts to 0.8 mile per second. For a mission time of 460 days, the velocity-increment requirement with all propulsive braking is 11.77 miles per second in 1965 compared with 11.2 miles per second in 1980.

Figure 14 indicates differences between the circular coplanar and three-dimensional data for 1980 of less than 3 percent (0.36 mile/sec) for mission times between 275 and 415 days. Thus, for preliminary mission studies, circular coplanar data could be used, which simplifies trajectory calculations. For more exact values, the effects of non-coplanar, elliptical orbits must be included.

The optimum 240-day trip departing in 1965 differs in many respects from the optimum 240-day trip in 1980. (See fig. 15.) The locations of the departure and arrival points (1, 2, 3, and 4) of the 1965 trip are different from those of the 1980 trip. These differences in location result in differences in the departure and arrival radii (R_1 , R_2 , R_3 , and R_4), path angles (α_1 , α_2 , α_3 , and α_4), and planet velocities (V_{q1} , V_{q2} , V_{q3} , and V_{q4}). It is these differences that result in the velocity-increment differences shown in figure 16. The 1980 values of

ΔV_1 , ΔV_2 , and ΔV_4 are higher than the 1965 values, whereas the 1965 value of ΔV_3 is higher than the 1980 value. Because of the synodic-period effect, the results presented herein for missions in 1979 and 1980 should be used only during preliminary planning of a particular Venus mission. An analysis for the particular synodic period of departure should be used for more exact results.

Round Trips for Wait Times Other Than 40 Days

In figure 17, velocity-increment parameters are plotted against the date of departure from Venus for a range of Venus-to-Earth trip time. These velocity increments (ΔV_3 , ΔV_4) along with those of figure 2 were used to calculate total velocity increments for missions having a wait time of 450 days.

Figures 18 to 23 indicate total velocity-increment requirements for round-trip missions from 460 to 780 days with a wait time in orbit at Venus of 200 days. Total velocity-increment requirements for a wait time of 390 days are indicated in figures 24, 25, and 26 for mission times of 580, 700, and 780 days, respectively. Total velocity increments for the same mission times but for wait times of 450 and 510 days are given in figures 27 to 32. For each of these wait and mission times, the total velocity-increment requirements for the three types of round trip are presented.

Effect of Wait Time

For mission times of 700 and 780 days, the minimum total velocity increments for the three types of round trip are plotted in figure 33 as a function of wait time. For these mission times, a wait time of 450 days resulted in near minimum velocity-increment requirements.

The envelope curves for the minimum total velocity-increment requirements shown in parts (a), (b), and (c) of figure 34 (one for each of the three types of round trip) indicate the effect of wait time on velocity increment for a range of mission times. For round-trip missions between 100 and about 630 days, increasing the wait time increases the velocity-increment requirements. For example, increasing the wait time from 40 to 200 days for a mission of 480 days increases the all propulsive-velocity increment from 11.4 to 16.5 miles per second (fig. 34(c)). For mission times between 660 and 780 days, however, increasing the wait time decreases the velocity-increment requirements. For example, increasing the wait time from 390 to 450 days decreases the all propulsive-velocity increment from 10.0 to 9.1 miles per second for a 760-day round trip. This trend is similar to the circular coplanar results of reference 1 where the optimum wait time is that associated with a minimum energy "Hohmann type" trip.

Effect of Atmospheric Braking

From figure 34, the minimum total velocity increment for each mission time can be obtained for each of the three types of round trip. These data are shown by the envelope curves of figure 35. As indicated by figure 35, atmospheric braking can be used to reduce the velocity-increment requirements for a given mission time or to reduce the mission time for a given velocity-increment requirement. To indicate the potential of atmospheric braking as a means of reducing mission time, a short-(460 days) and a long- (760 days) duration trip are arbitrarily chosen for comparison. Employing all propulsive braking, the 460- and 760-day trips require total velocity increments of 11.2 and 9.05 miles per second, respectively. With atmospheric braking at Earth and Venus, these values become 4.7 and 4.1 miles per second. Thus, the increase in velocity increment that accompanies a reduction in trip time diminishes when atmospheric braking is employed. This trend may be significant in deciding between a long- or short-duration round trip. For a velocity-increment requirement of 12 miles per second, atmospheric braking at Venus and Earth can be used to reduce the mission time from 400 to 195 days. For missions between 400 and 600 days, a variation of 0.8 mile per second or less in total velocity increment exists for each of the three types of round trip.

Arrival Velocity

When missions utilizing atmospheric braking are planned, the velocity of arrival at the target planet must be known in order to study re-entry conditions. Velocities were computed from the expression

$$v_A^2 = v_H^2 + \frac{2\mu}{r} \quad (3)$$

where r is the planet's radius. The velocity at the "top" of the atmosphere is negligibly different from these values.

Figure 36 indicates arrival velocities at Earth for wait times at Venus of 40 and 450 days. The arrival velocities of figure 36(a) are for minimum values of $\Delta V_1 + \Delta V_3$ and mission times ranging from 150 to 780 days. The curves of figure 36(b) are based on minimum values of velocity increment where atmospheric braking has been used on arrival at Earth only. As the mission time is decreased from 400 to 150 days, the arrival velocities of figure 36(a) decrease slightly (8.7 to 8.5 miles/sec), whereas the arrival velocities of figure 36(b) increase from 8.91 to 12.7 miles per second.

Figure 37 indicates arrival velocities at Venus for a range of mission time and for wait times of 40 and 450 days. For each mission time, the value of $\Delta V_1 + \Delta V_3$ has been minimized. For a wait time of 40 days at Venus, the arrival velocity increased from 7.3 to 19 miles per second as the mission time decreased from 530 to 150 days.

Undoubtedly a limiting approach speed exists beyond which atmospheric braking is impractical. The limiting speed would be dependent on thermodynamic considerations, human tolerance, and guidance accuracy. Thus, a combination of atmospheric and propulsive braking may be required. If, as in reference 6, an approach speed of 10 miles per second is possible, however, atmospheric braking instead of propulsive braking on arrival at Earth would thus be possible for mission times between 260 and about 670 days for a wait time of 40 days. Similarly, atmospheric braking at Venus and Earth would be possible for mission times between 350 and 630 days (figs. 36(a) and 37).

SUMMARY OF RESULTS

The dependence of velocity increment on mission time was such that a number of local minimum values of velocity increment occurred for mission times between 100 and 780 days. One of these minimums was for a mission time of about 460 days and a wait time of 40 days. Another, lower, minimum occurred for a mission time of about 760 days and a wait time of 450 days. When all propulsive braking was employed, the total velocity increments at these minimums were 11.2 and 9.05 miles per second, respectively. The values became 4.7 and 4.1 when atmospheric braking was employed at both Venus and Earth. Thus, the increase in velocity increment that accompanied a reduction in trip time diminished when atmospheric braking was employed. This trend might be significant in deciding between a long or short round-trip mission to Venus. In addition to reducing the velocity-increment requirements for a fixed mission time, atmospheric braking can be used to reduce mission time at a fixed velocity-increment requirement. For a velocity increment of 12 miles per second, atmospheric braking used on arrival at Venus and Earth reduced the mission time from 400 to 195 days.

The dependence of velocity increment on wait time at Venus was such that for mission times between 100 and 630 days, increasing the wait time increased the velocity-increment requirement. For mission times between 630 and 780 days, increasing the wait time decreased the velocity-increment requirement.

For preliminary planning of round-trip missions to Venus, the effect of synodic period of departure on velocity-increment requirements may be neglected. For more exact numbers, however, an analysis for the particular synodic period of departure should be used. For a mission of 460 days, the velocity increment requirement with all propulsive braking was 11.77 miles per second in 1965 compared with 11.2 in 1980.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, June 4, 1962

APPENDIX - SYMBOLS

D	departure date from orbit at 1.1 planet radii
P	perihelion
R ₁	heliocentric radius of vehicle on date of departure from Earth, miles
R ₂	heliocentric radius of vehicle on date of arrival at Venus, miles
R ₃	heliocentric radius of vehicle on date of departure from Venus, miles
R ₄	heliocentric radius of vehicle on date of arrival at Earth, miles
r	planet radius, miles
T _m	orbit-to-orbit round-trip mission time between Earth and Venus, days
T _w	wait time in orbit at Venus, days
T _{⊕♀}	orbit-to-orbit trip time from Earth to Venus, days
T _{♀⊕}	orbit-to-orbit trip time from Venus to Earth, days
V _A	arrival velocity at surface of planet, miles/sec
V _H	hyperbolic velocity of vehicle, miles/sec
V _{⊕1}	velocity of Earth on date of departure, miles/sec
V _{♀2}	velocity of Venus on date of arrival, miles/sec
V _{♀3}	velocity of Venus on date of departure, miles/sec
V _{⊕4}	velocity of Earth on date of arrival, miles/sec
ΔV ₁	velocity increment to depart orbit at 1.1 Earth radii, miles/sec
ΔV ₂	velocity increment to establish orbit at 1.1 Venus radii, miles/sec
ΔV ₃	velocity increment to depart orbit at 1.1 Venus radii, miles/sec
ΔV ₄	velocity increment to establish orbit at 1.1 Earth radii, miles/sec

- α_1 heliocentric path of vehicle with respect to local horizontal on date of departure from Earth, deg
- α_2 heliocentric path of vehicle with respect to local horizontal on date of arrival at Venus, deg
- α_3 heliocentric path of vehicle with respect to local horizontal on date of departure from Venus, deg
- α_4 heliocentric path of vehicle with respect to local horizontal on date of arrival at Earth, deg
- λ heliocentric latitude with respect to plane of the ecliptic, deg
- θ transfer plane travel angle from departure radius to arrival radius, deg
- μ gravitational force constant of planet, miles³/sec²

Subscripts:

\oplus Earth

\ominus Venus

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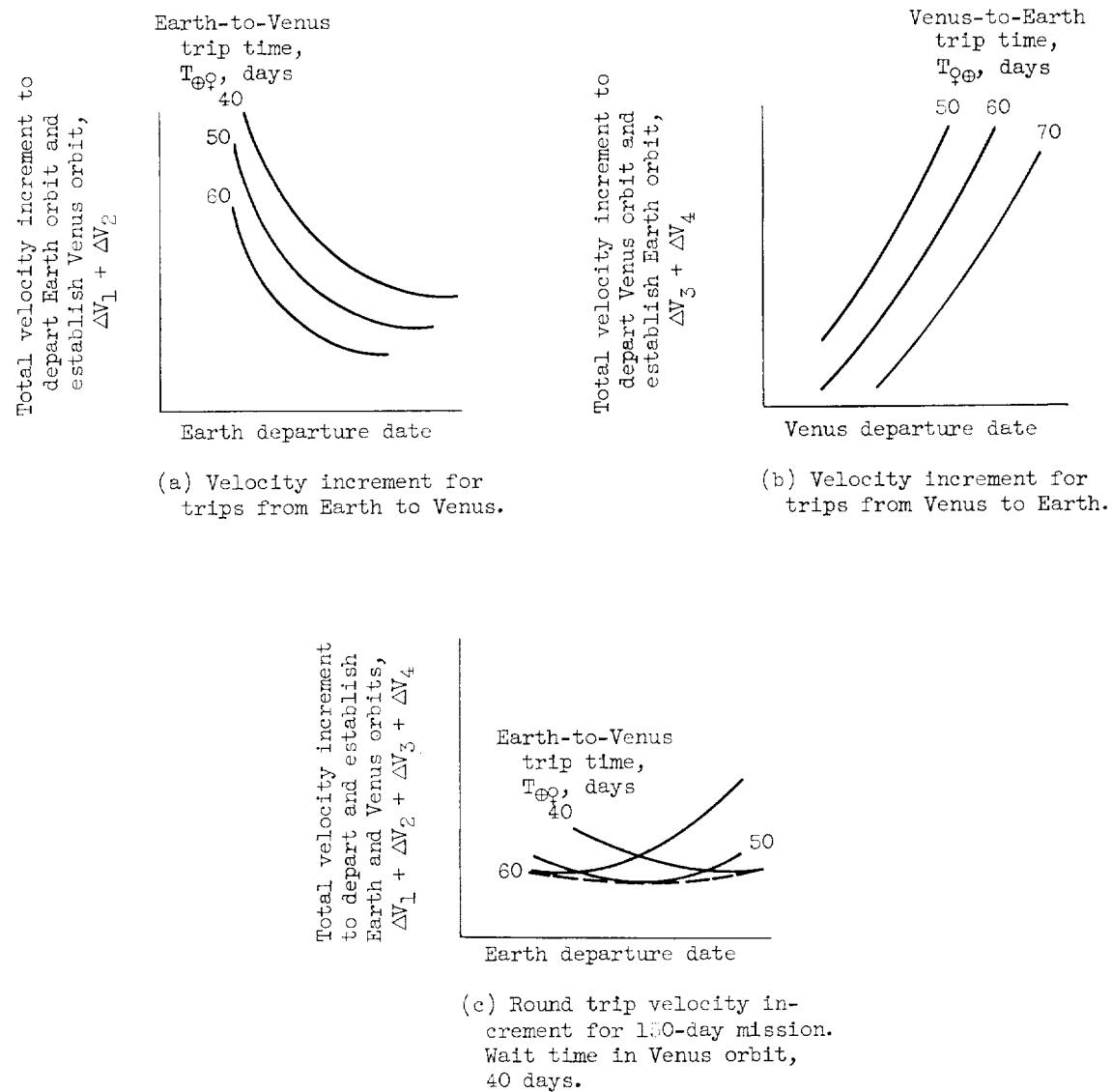
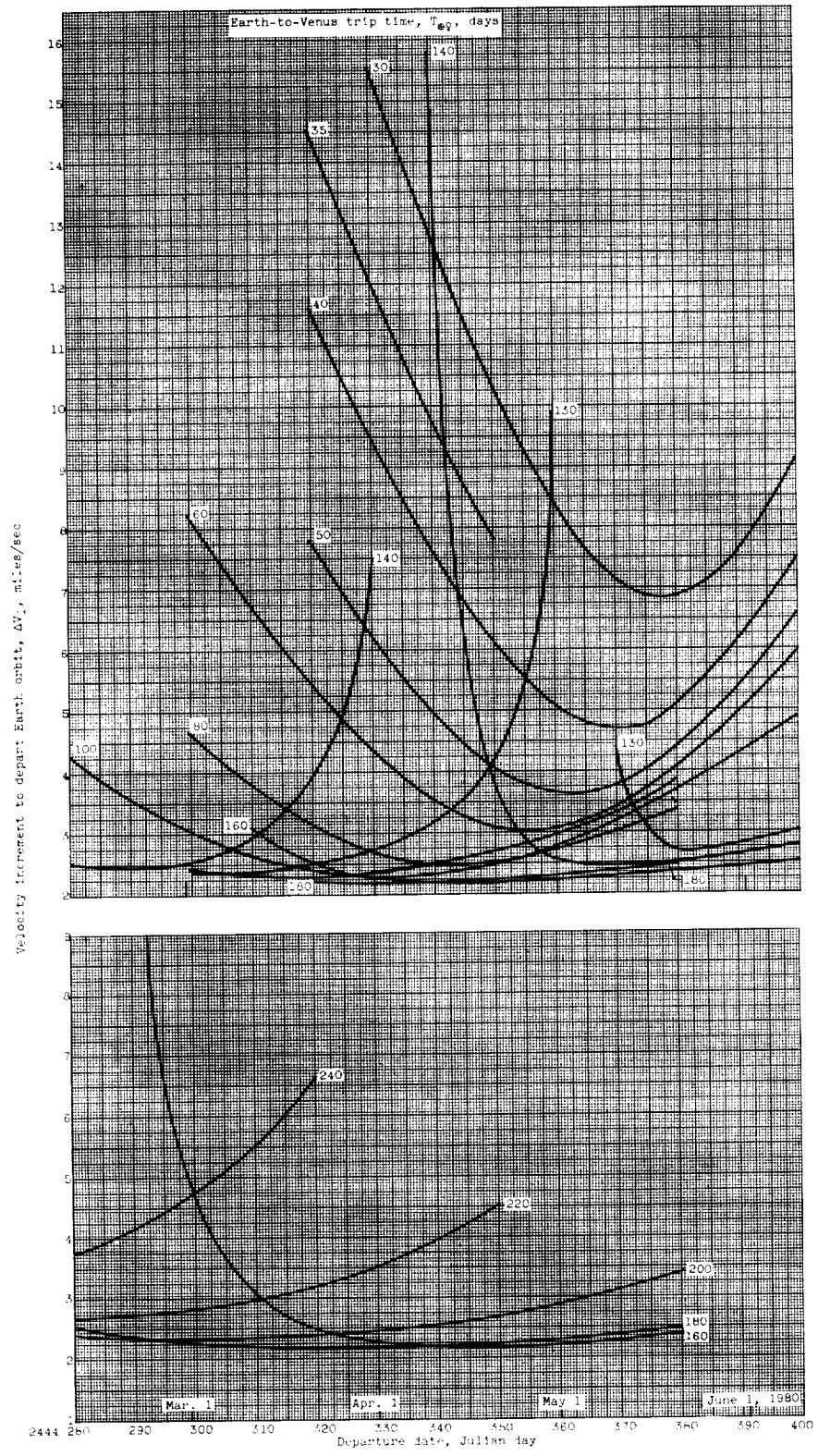
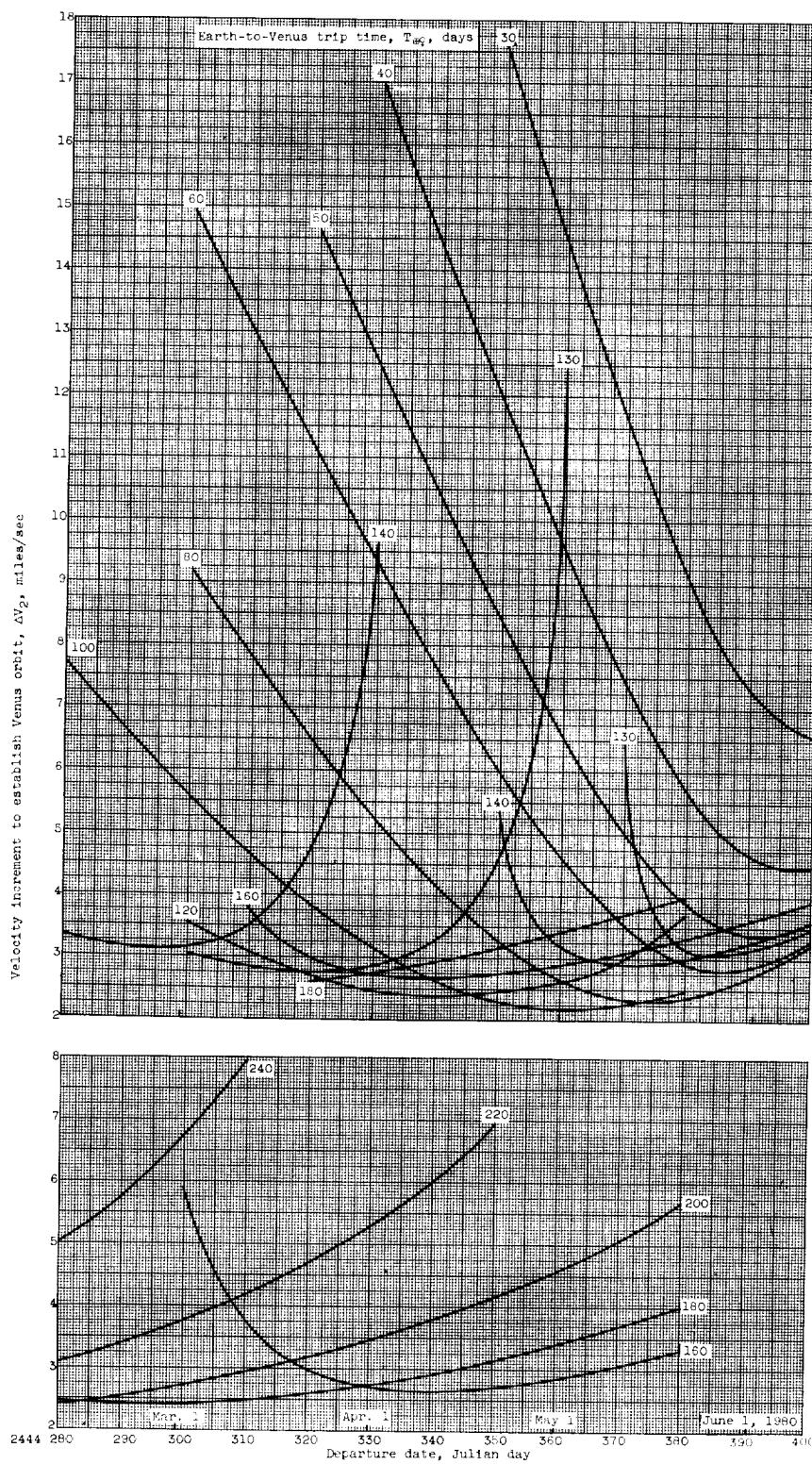


Figure 1. - Procedure for optimizing total velocity increment for round trip to Venus given mission time and wait time.



(a) Depart Earth.

Figure 2. - Velocity increments for trips from Earth to Venus starting and ending in circular orbit at 1.1 planet radii, 1979-80.



(b) Arrive Venus.

Figure 2. - Concluded. Velocity increments for trips from Earth to Venus starting and ending in circular orbit at 1.1 planet radii, 1979-80.

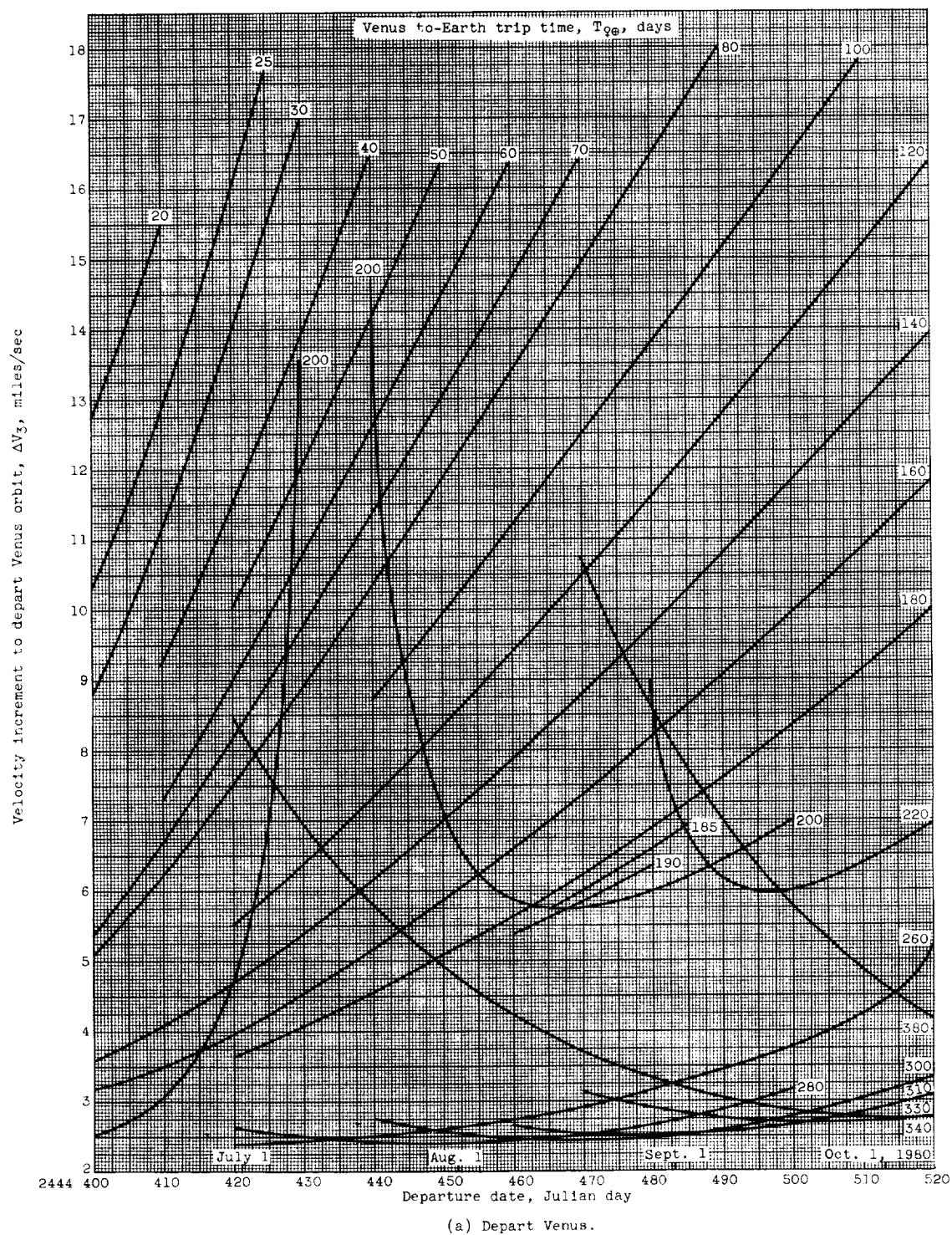


Figure 3. - Velocity increments for trips from Venus to Earth starting and ending in circular orbit at 1.1 planet radii, 1980-81.

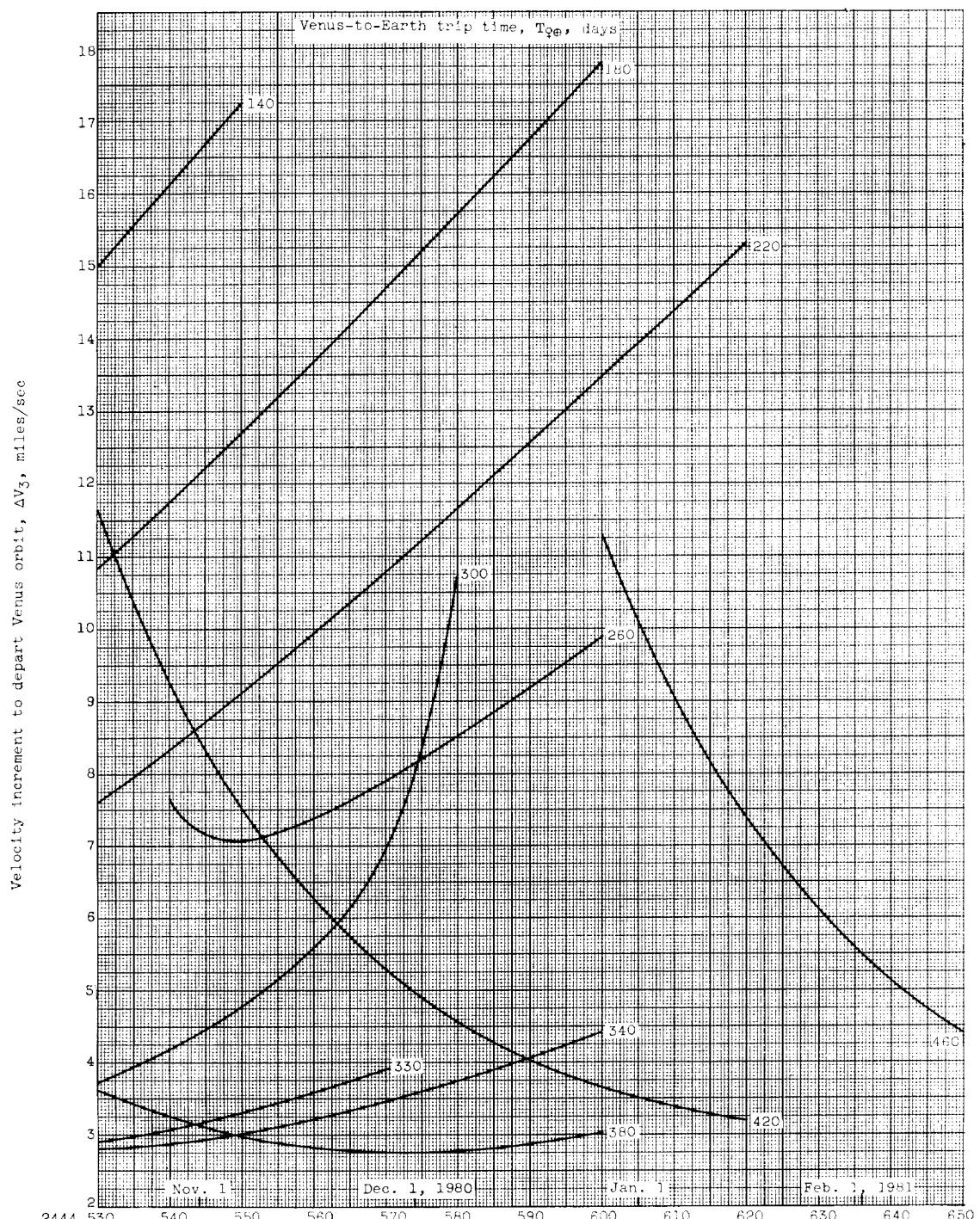


Figure 3. - Continued. Velocity increments for trips from Venus to Earth starting and ending in circular orbit at 1.1 planet radii, 1980-81.

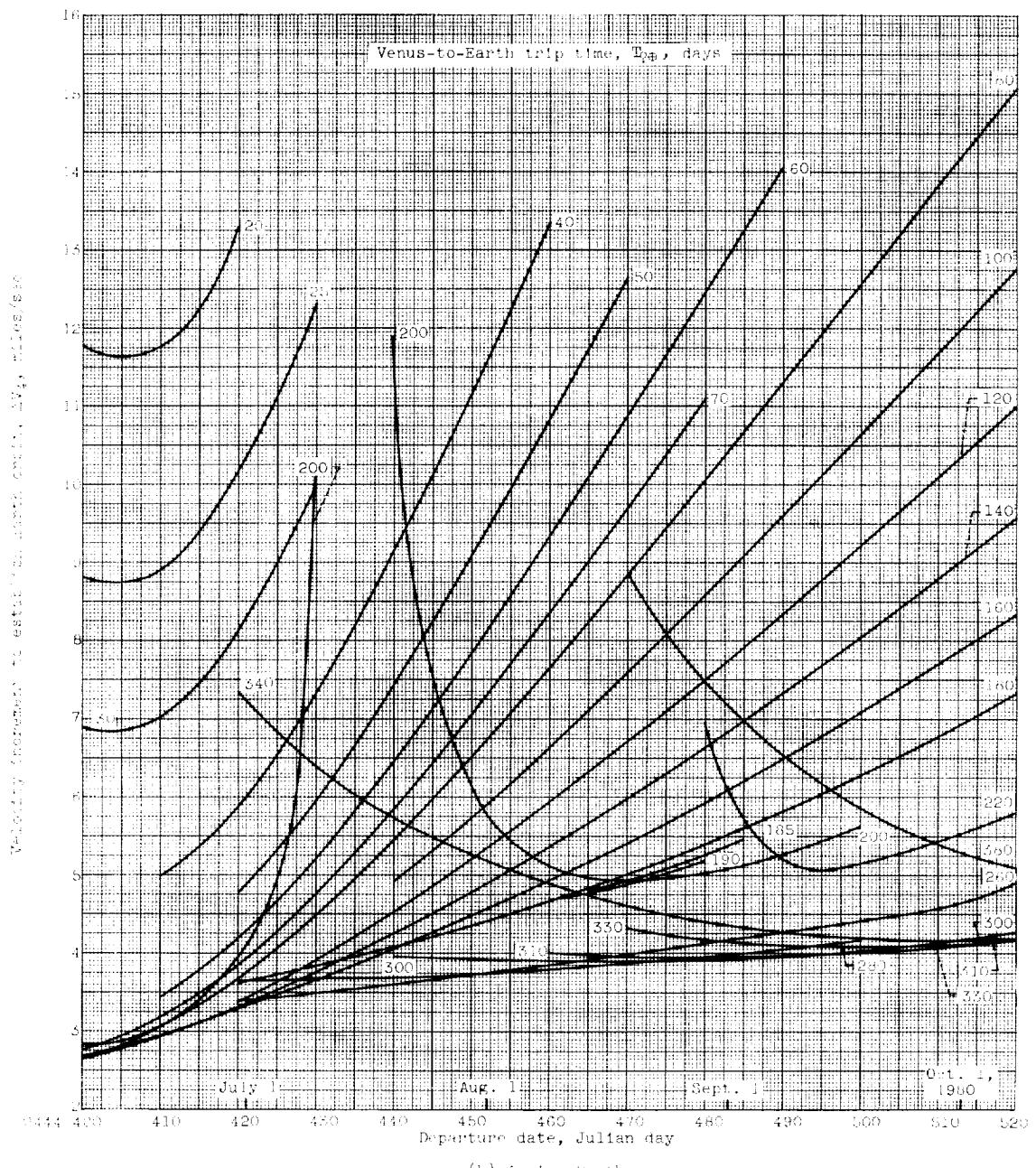
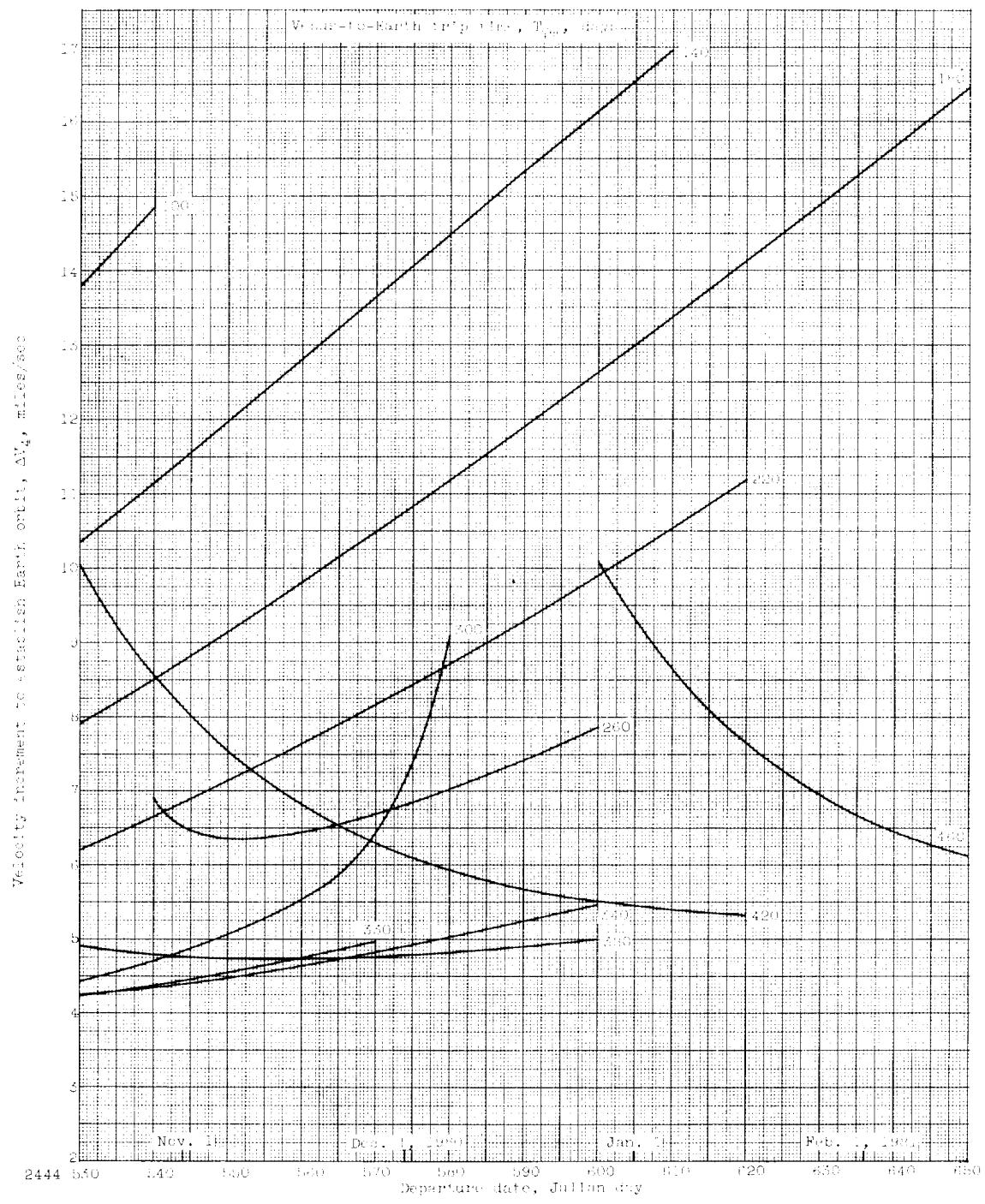
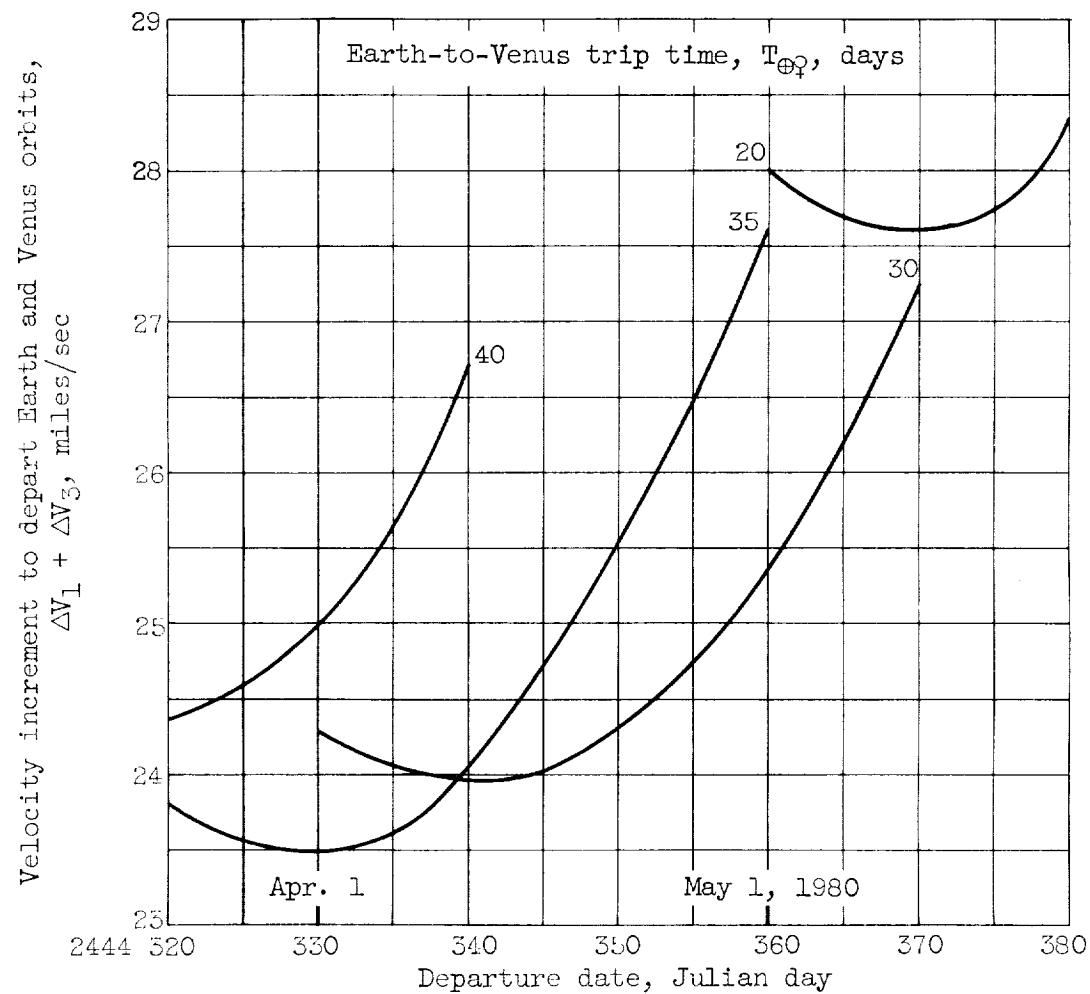


Figure 3. - Continued. Velocity increments for trips from Venus to Earth starting and ending in circular orbit at 1.1 planet radii, 1960-61.



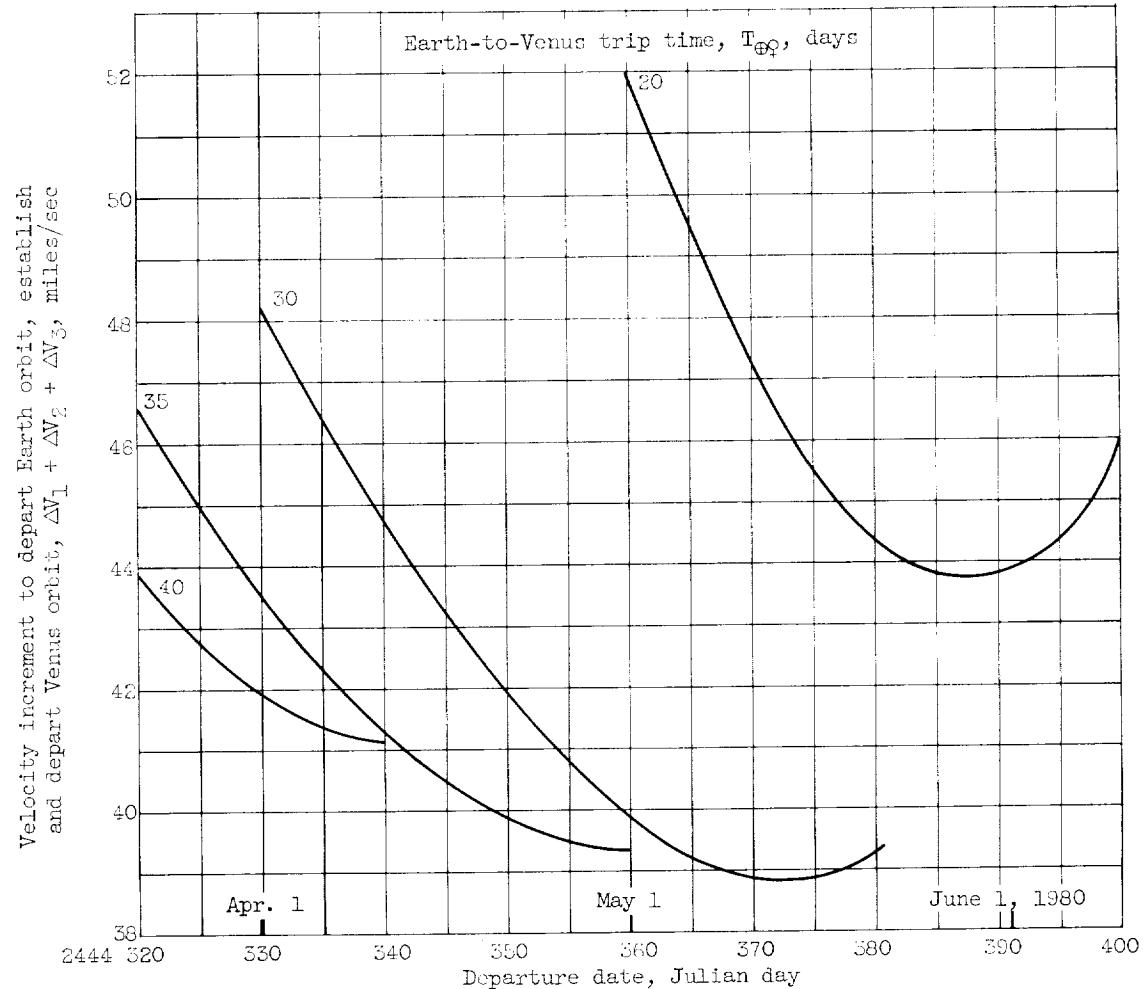
(b) Concluded. Arrive Earth.

Figure 8. - Declined. Velocity increments for trips from Venus to Earth starting and ending in circular orbit at 1.1 planet radii, January.



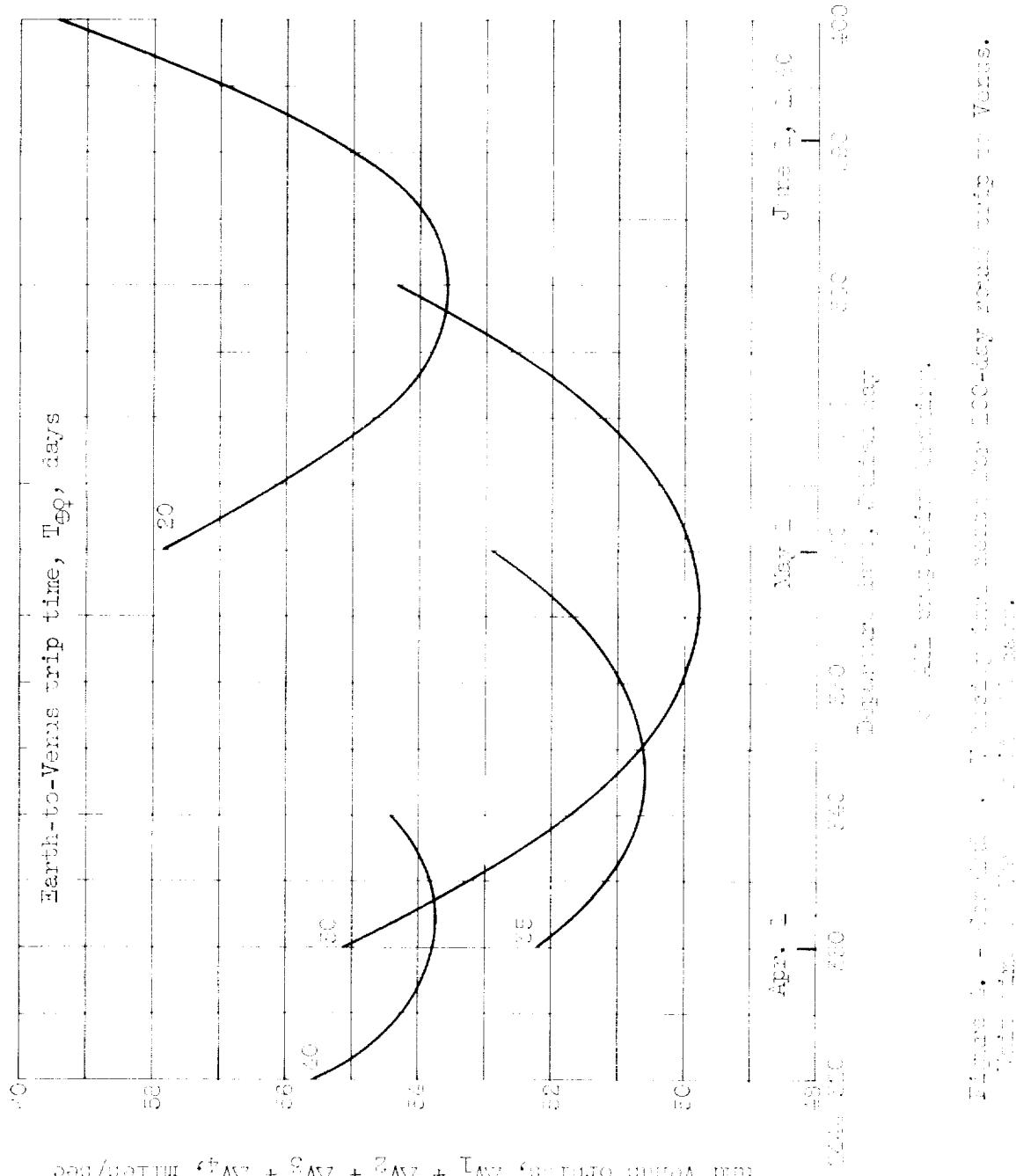
(a) Atmospheric braking at Venus and Earth.

Figure 4. - Velocity increments for 100-day round trip to Venus. Wait time in Venus orbit, 40 days.



(b) Atmospheric braking at Earth.

Figure 4. - Continued. Velocity increments for 100-day round trip to Venus.
Wait time in Venus orbit, 40 days.



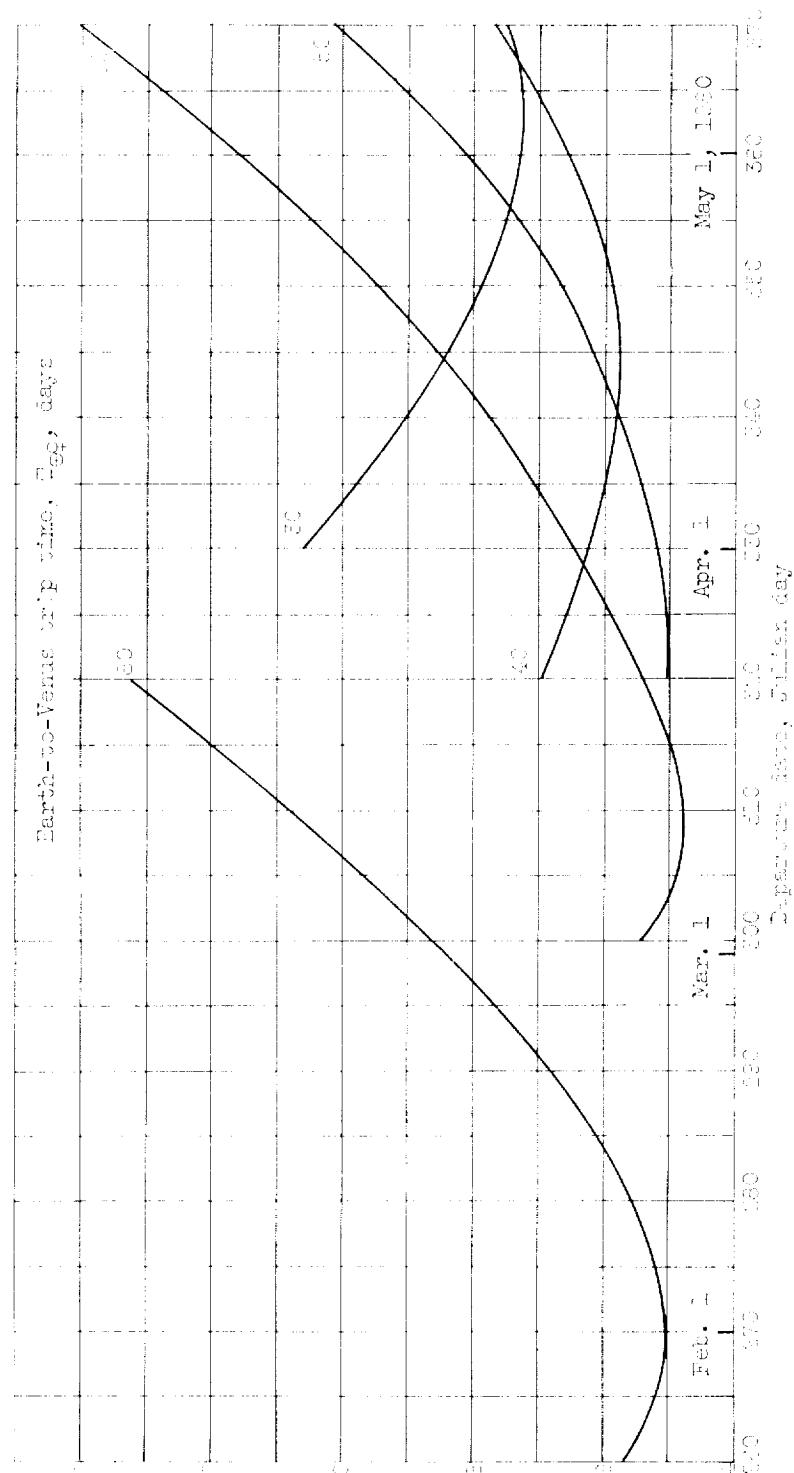
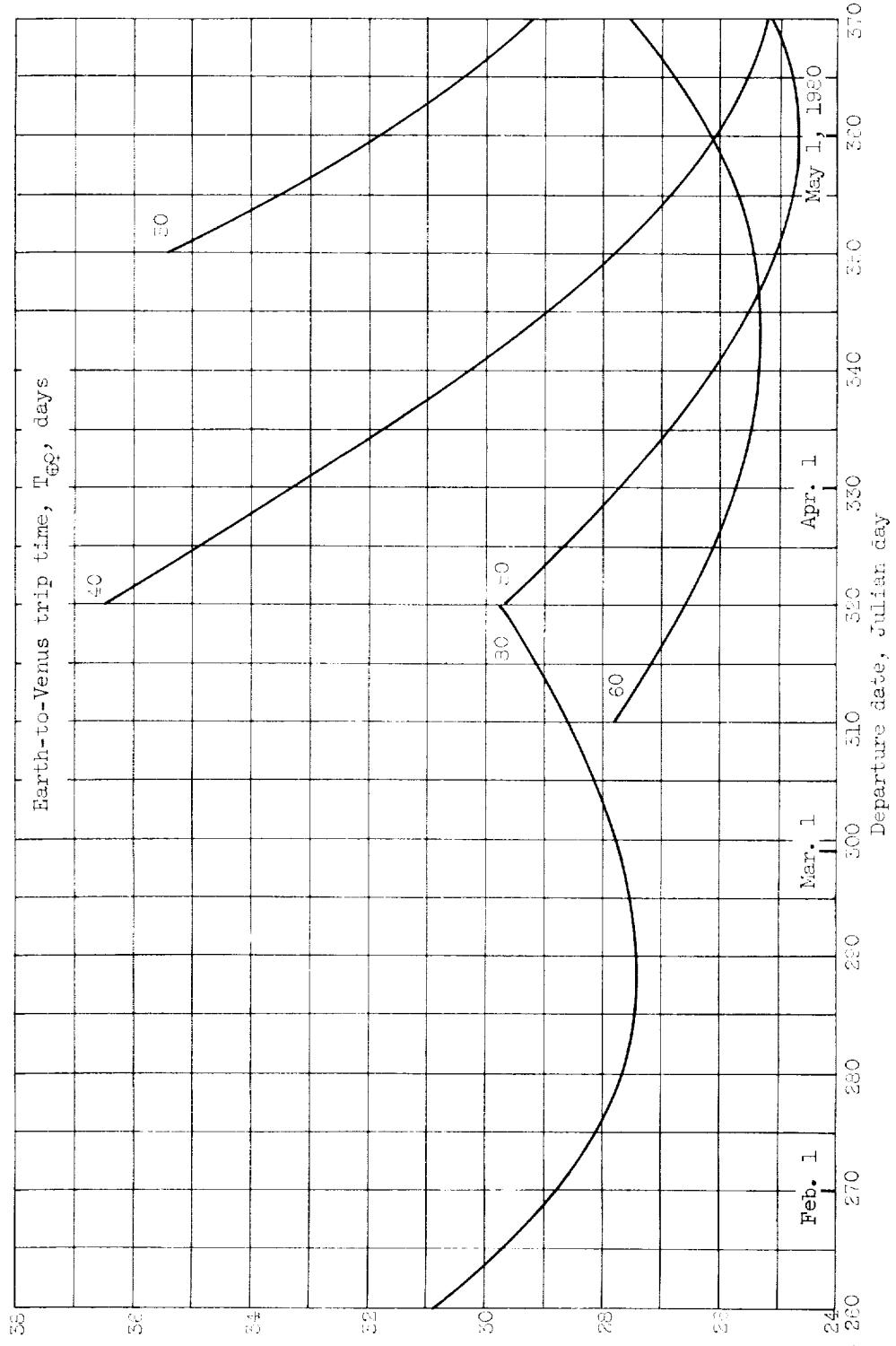
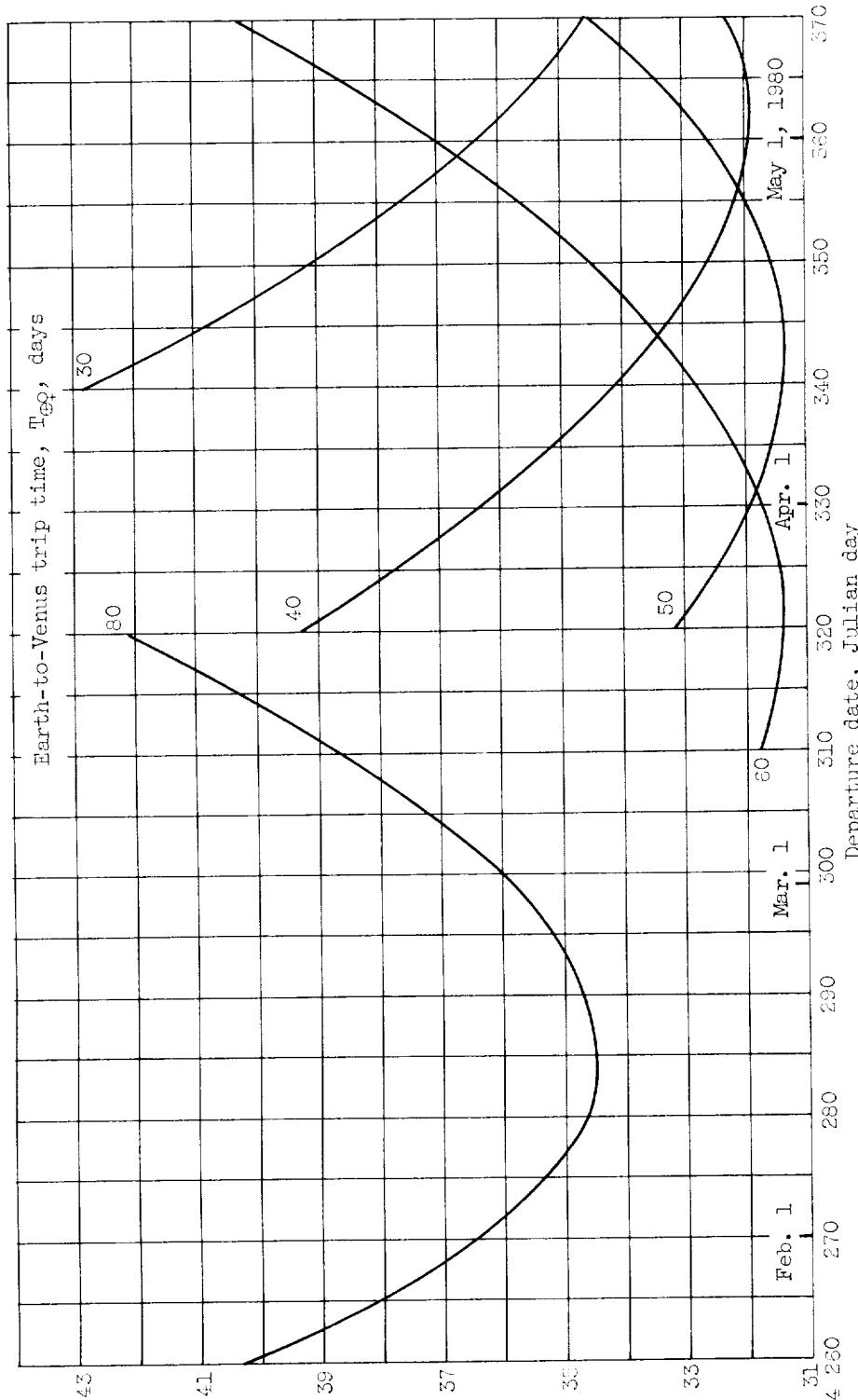


Figure 3 - Velocity trajectories from May 1, 1960, to March 1, 1961, for Venus.



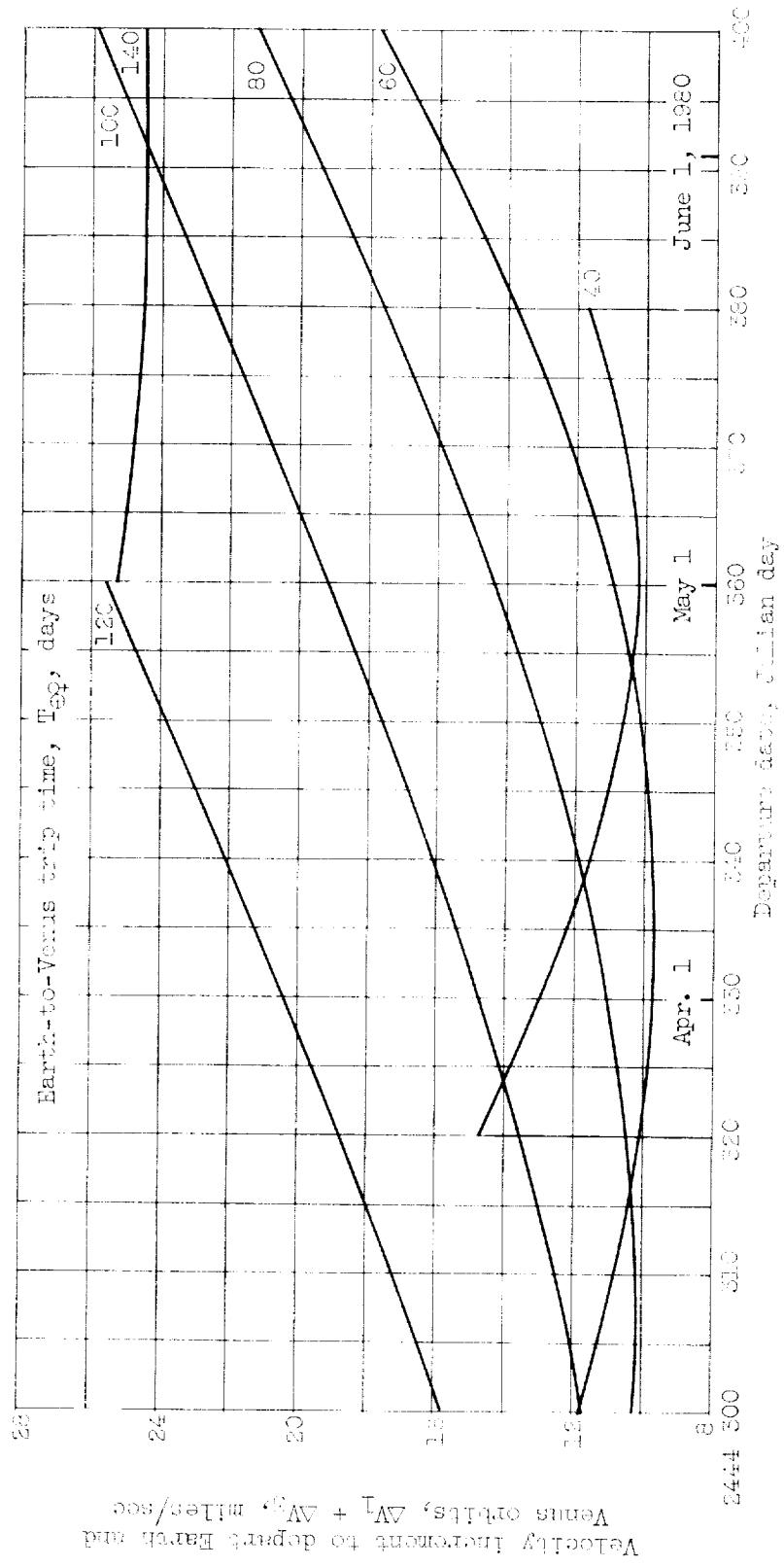
(b) Atmospheric braking at Earth.

Figure 5. - Continued. Velocity increments for 10-day round trip to Venus. Wait time in Venus orbit, 40 days.



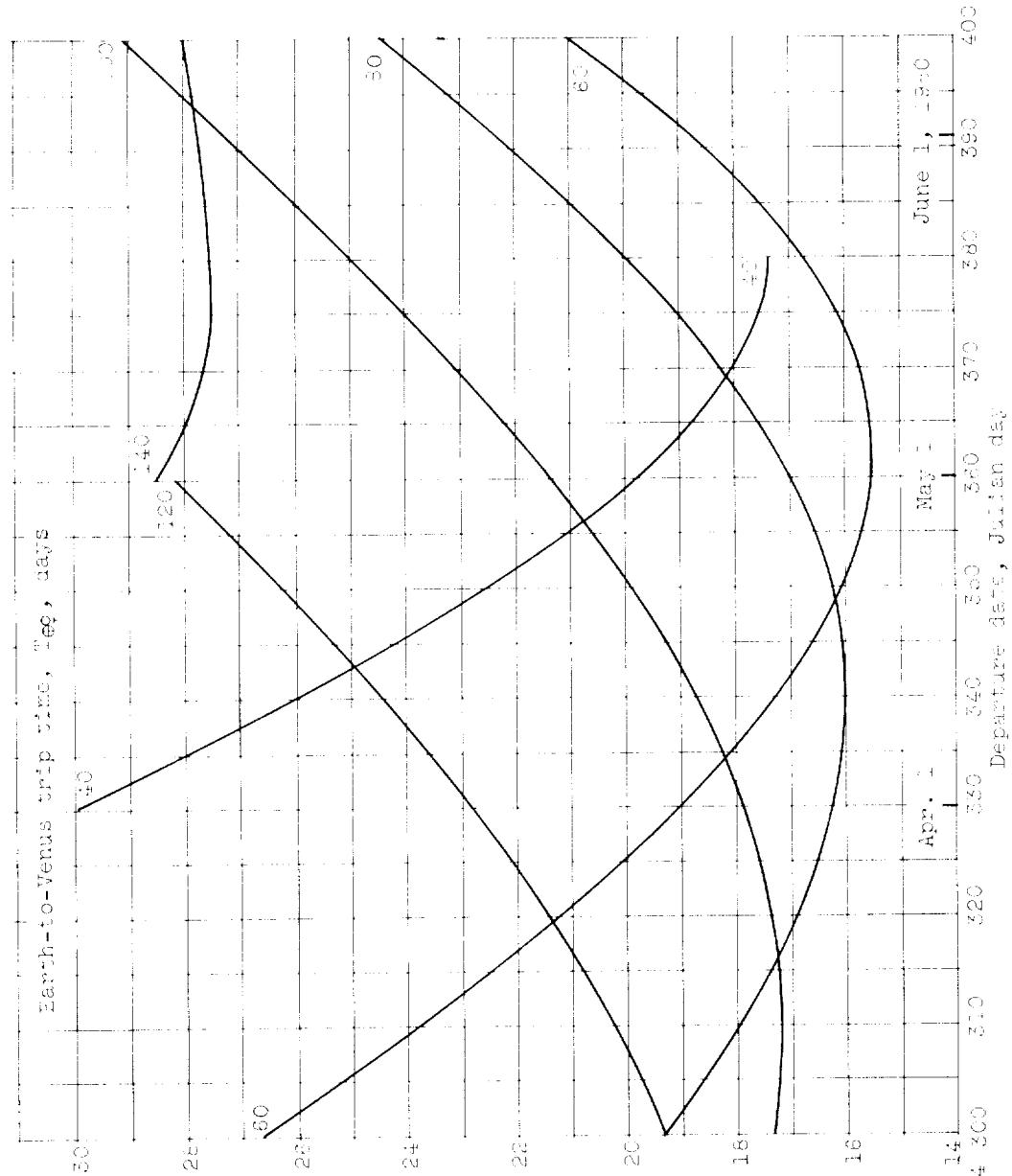
(c) All propulsive braking.

Figure 5. - Concluded. Velocity increments for 150-day round trip to Venus. Wait time in Venus orbit, 40 days.

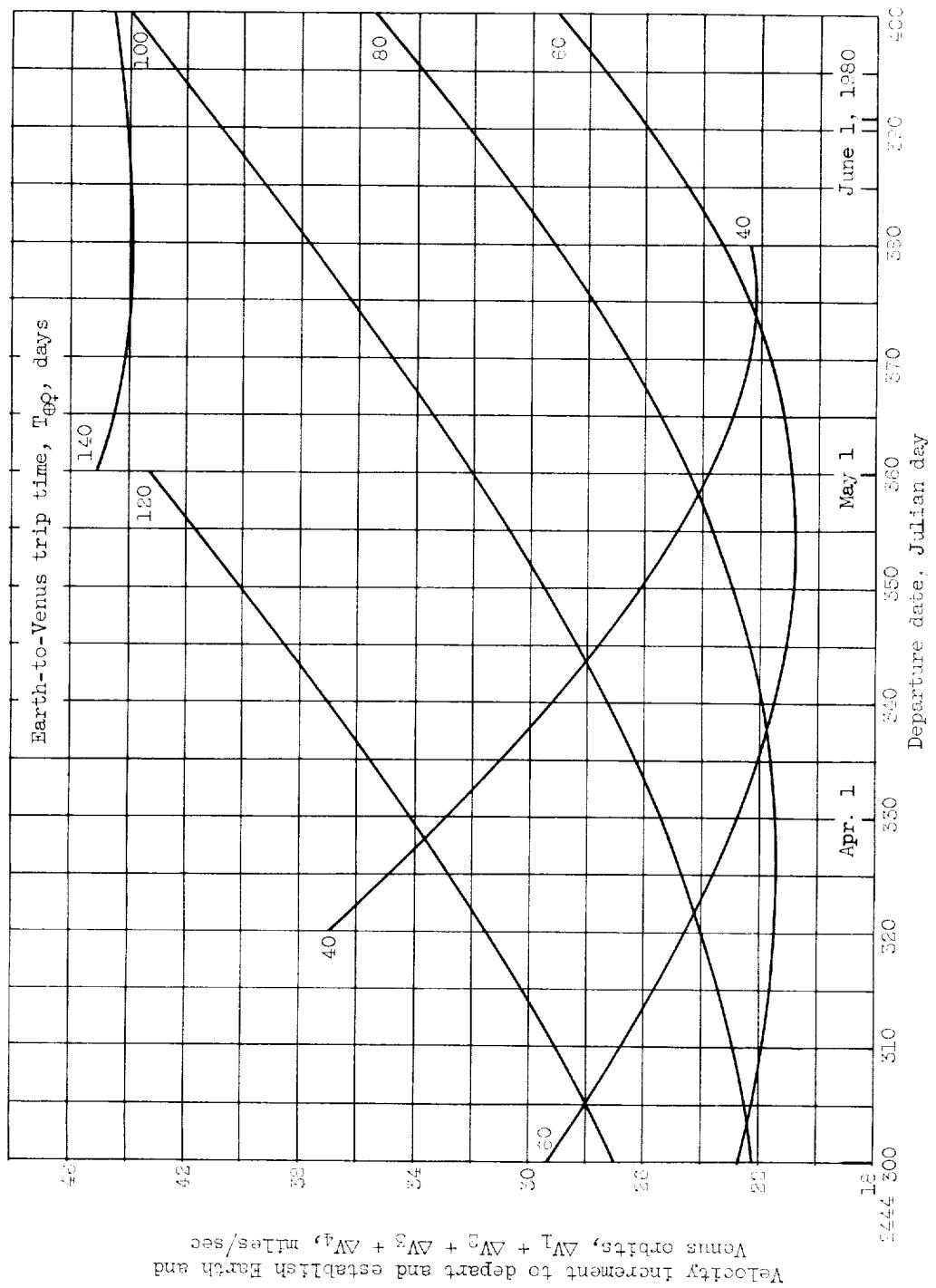


Atmospheric braking at Venus and return.

Figure 3. - Velocity increments for 240-day round trip to Venus. Wait time in Venus orbit, ± 10 days.

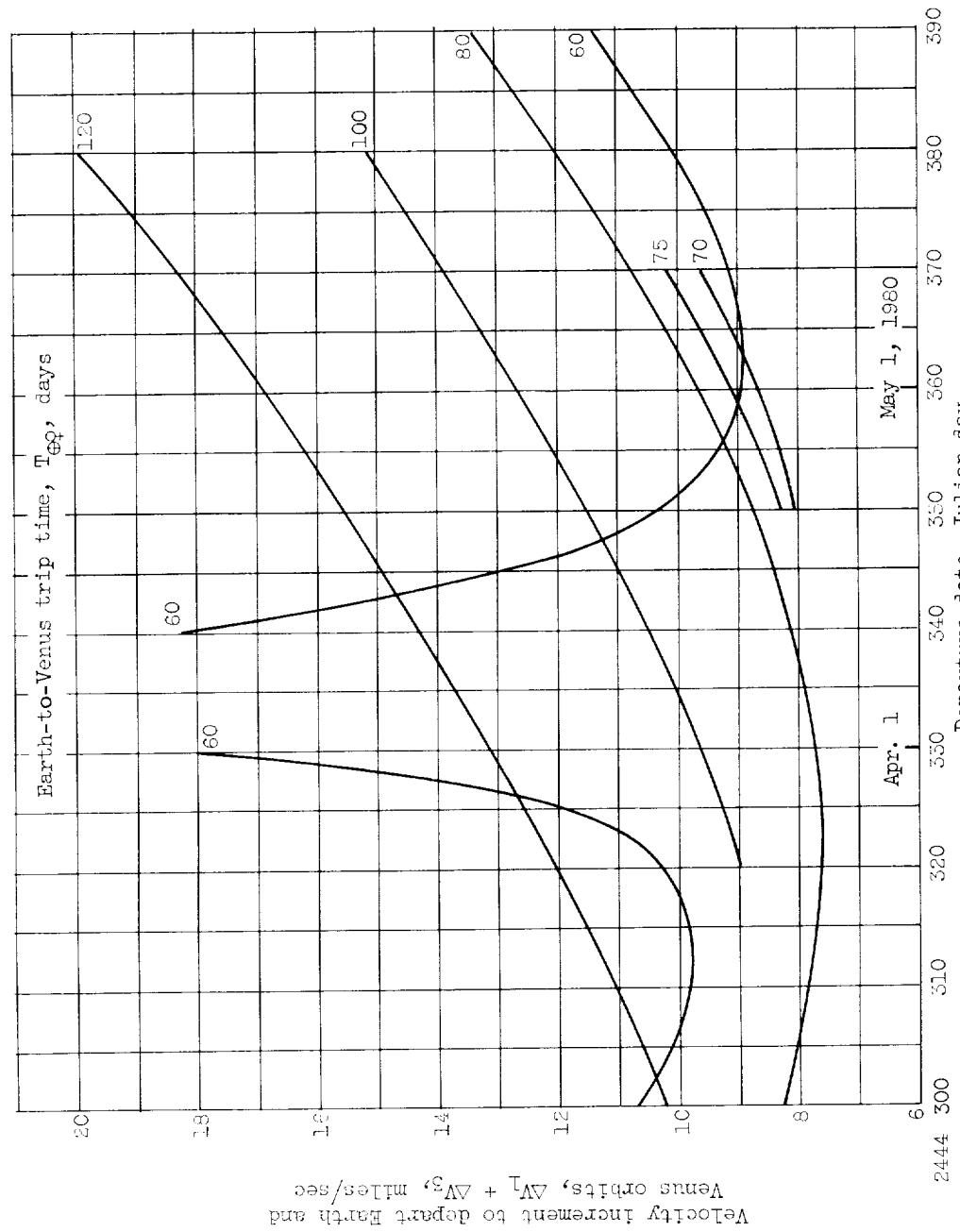


(b) Atmosphere crossing at each
stage - Continued. Velocity increment for 240-day round trip to Venus.
Same time in Venus orbit, 40 deg.



(c) All propulsive braking.

Figure 6. - Concluded. Velocity increments for 240-day round trip to Venus. Wait time in Venus orbit, 40 days.



(a) Atmospheric braking at Venus and Earth.

Figure 7. - Velocity increments for 300-day round trip to Venus. Wait time in Venus orbit, 40 days.

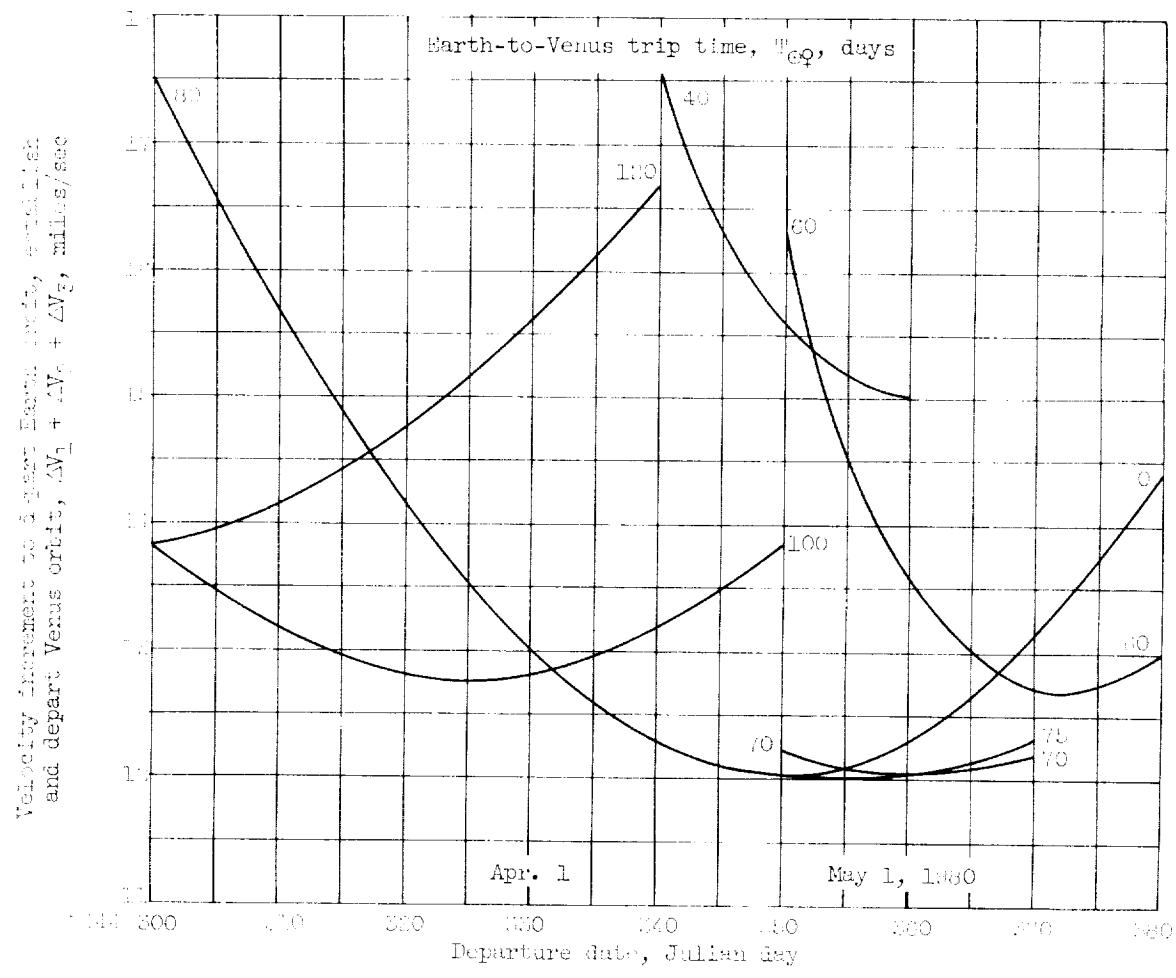


Figure 7. - Continued. Velocity increments for 300-day round trip to Venus.
Wait time in Venus orbit, 40 days.

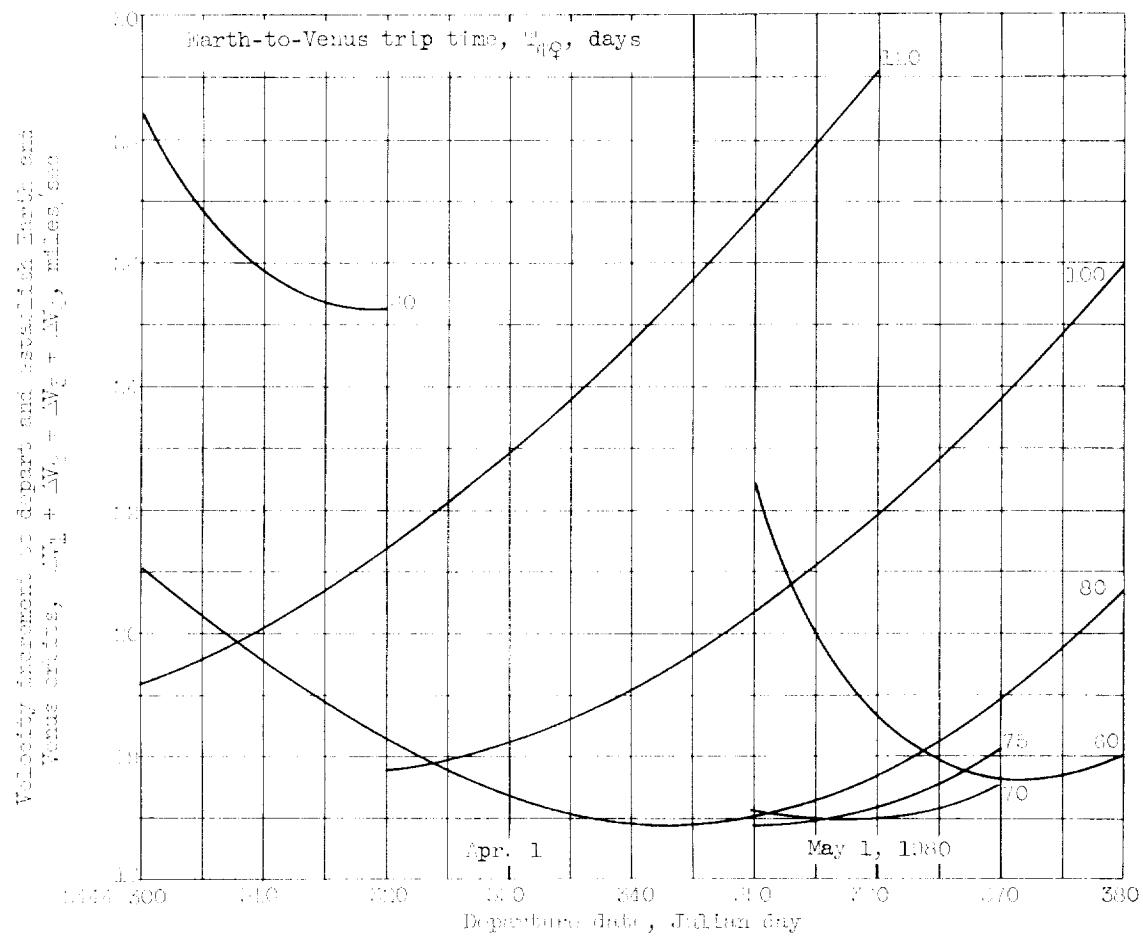


Figure 7. - Concluded. Velocity increments for 300-day round trip to Venus. Wait time in Venus orbit, 40 days.

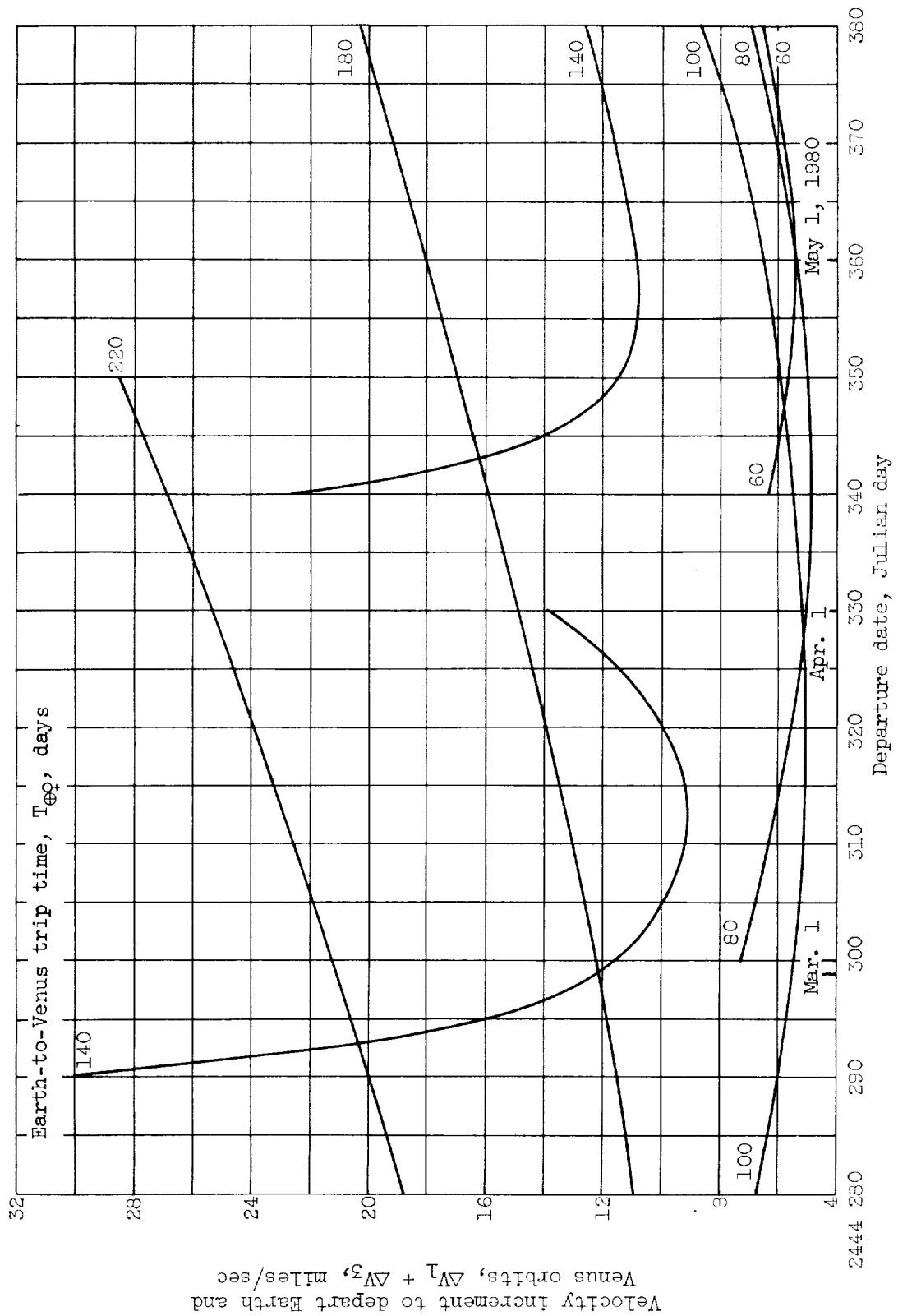
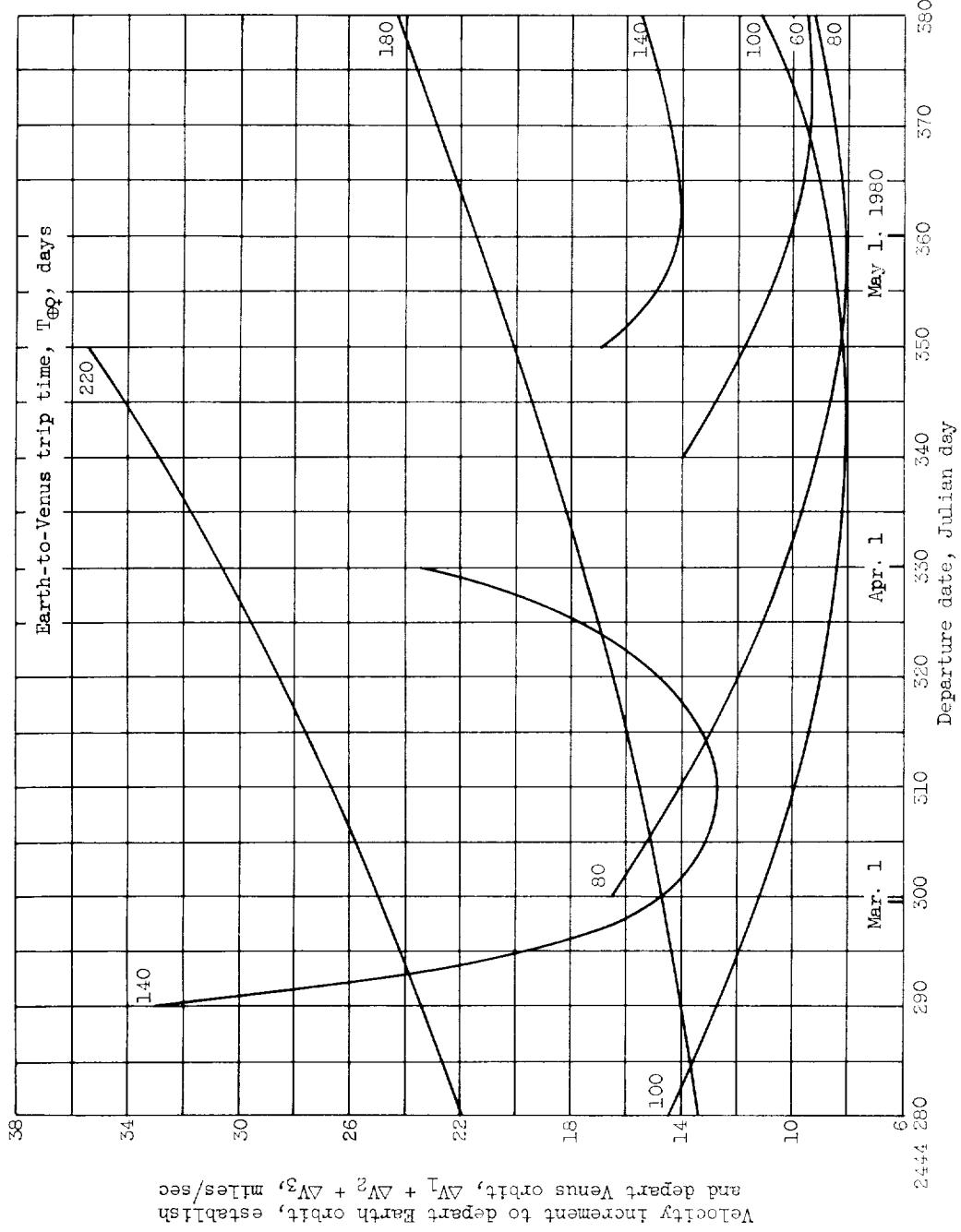


Figure 8. - Velocity increments for 400-day round trip to Venus. Wait time in Venus orbit, 40 days.



(b) Atmospheric braking at Earth.

Figure 9. - Continued. Velocity increments for 400-day round trip to Venus. Wait time in Venus orbit, 40 days.

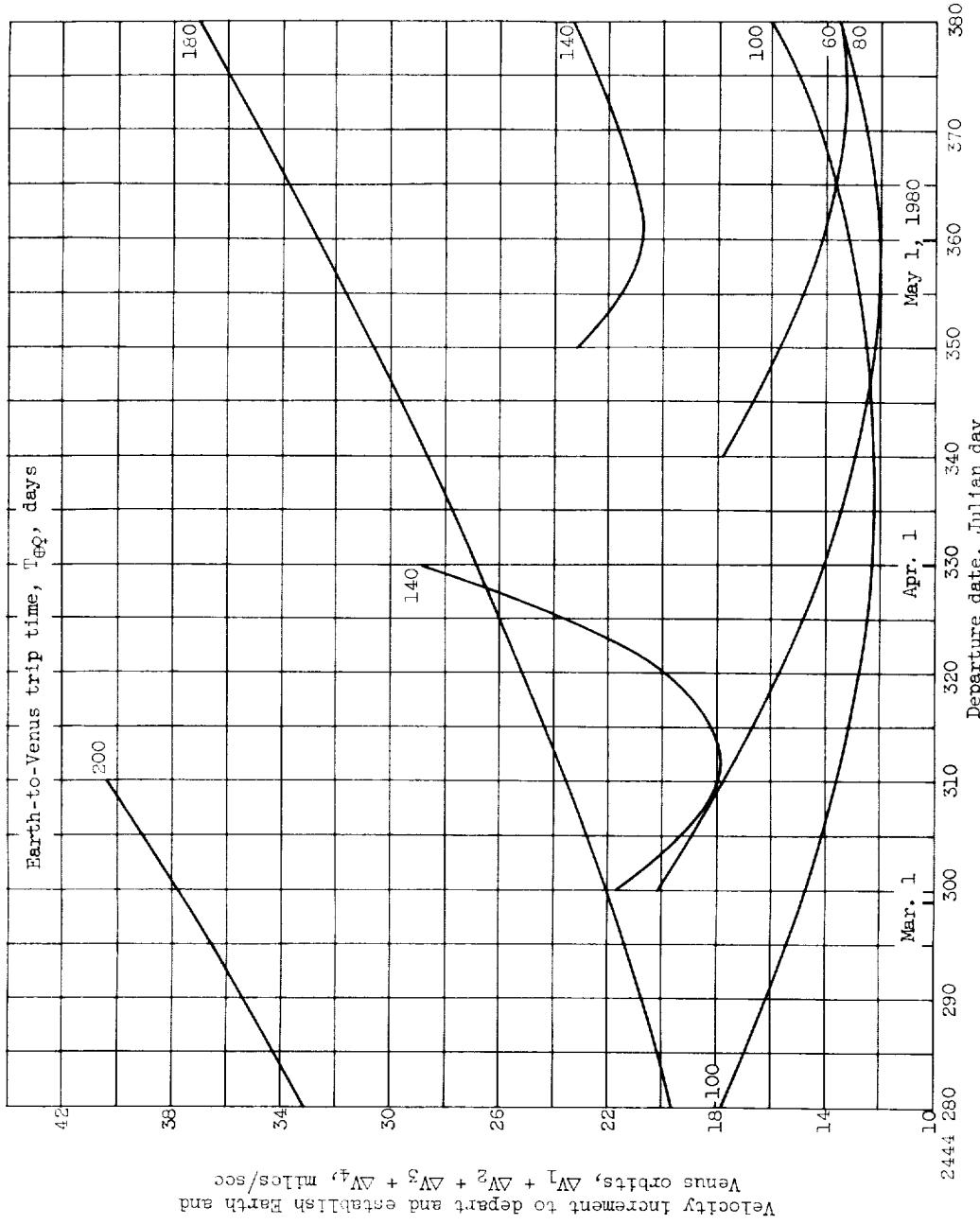
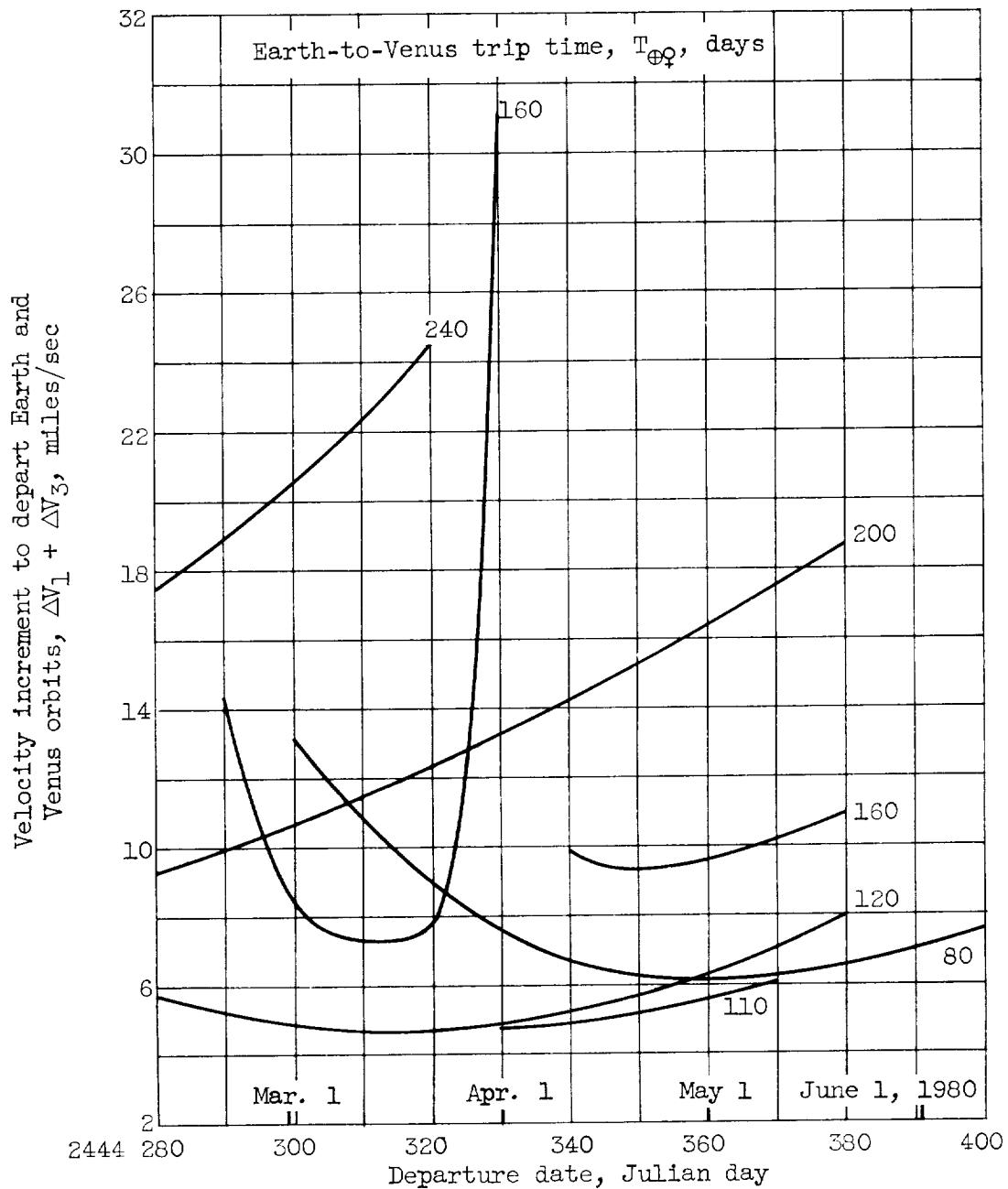
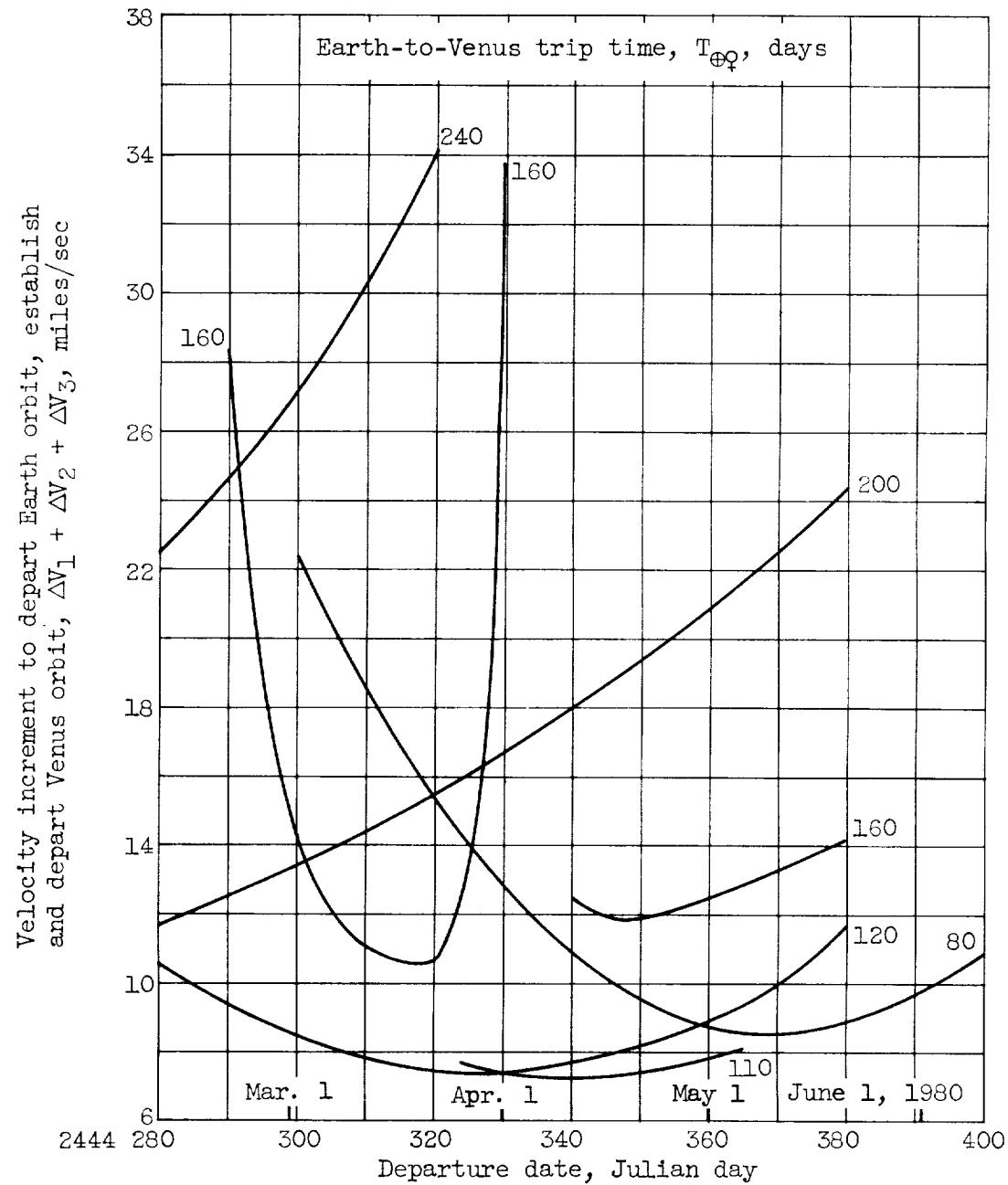


Figure 8. - Concluded. Velocity increments for 400-day round trip to Venus. Wait time in Venus orbit, 40 days.



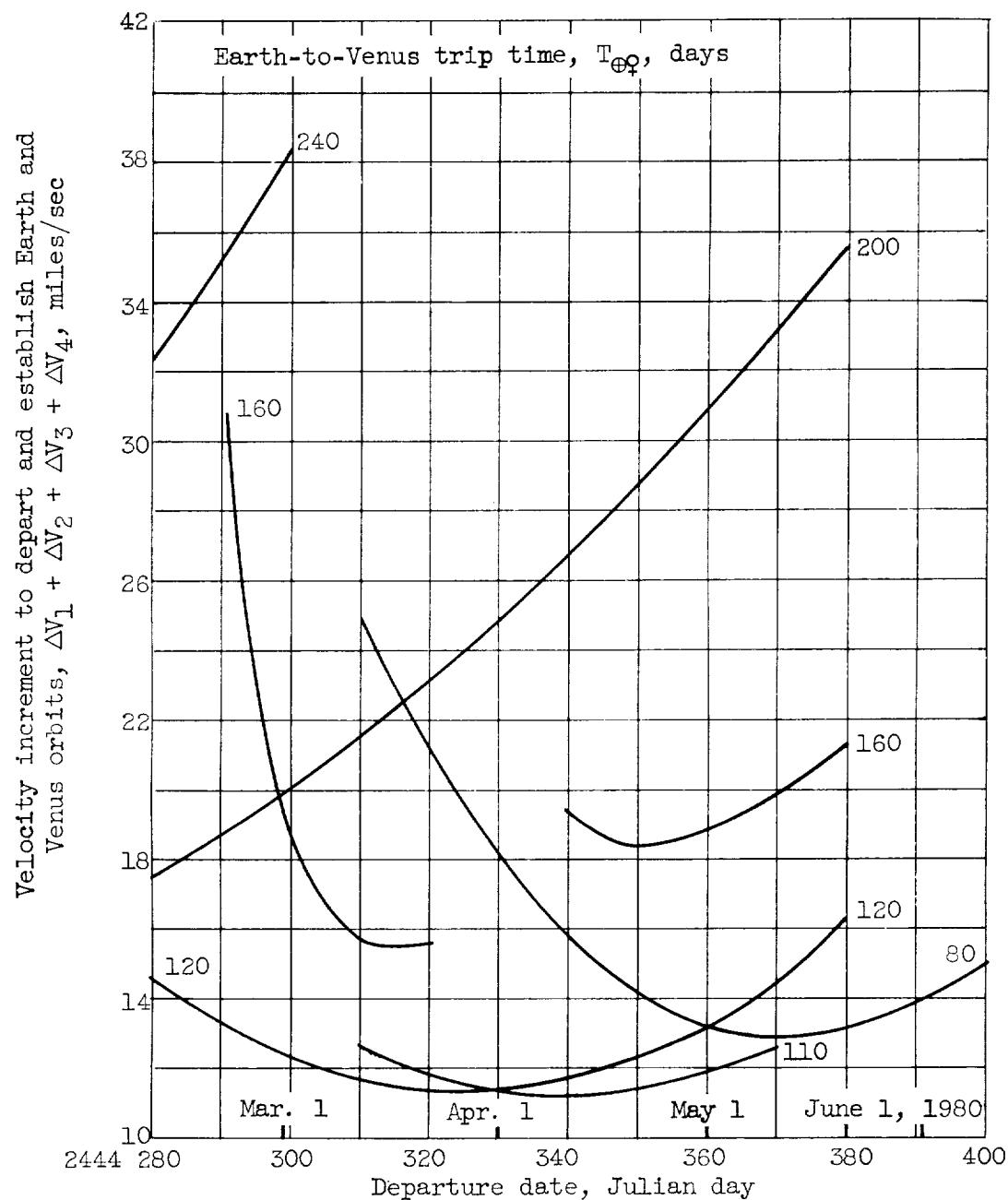
(a) Atmospheric braking at Venus and Earth.

Figure 9. - Velocity increments for 460-day round trip to Venus. Wait time in Venus orbit, 40 days.



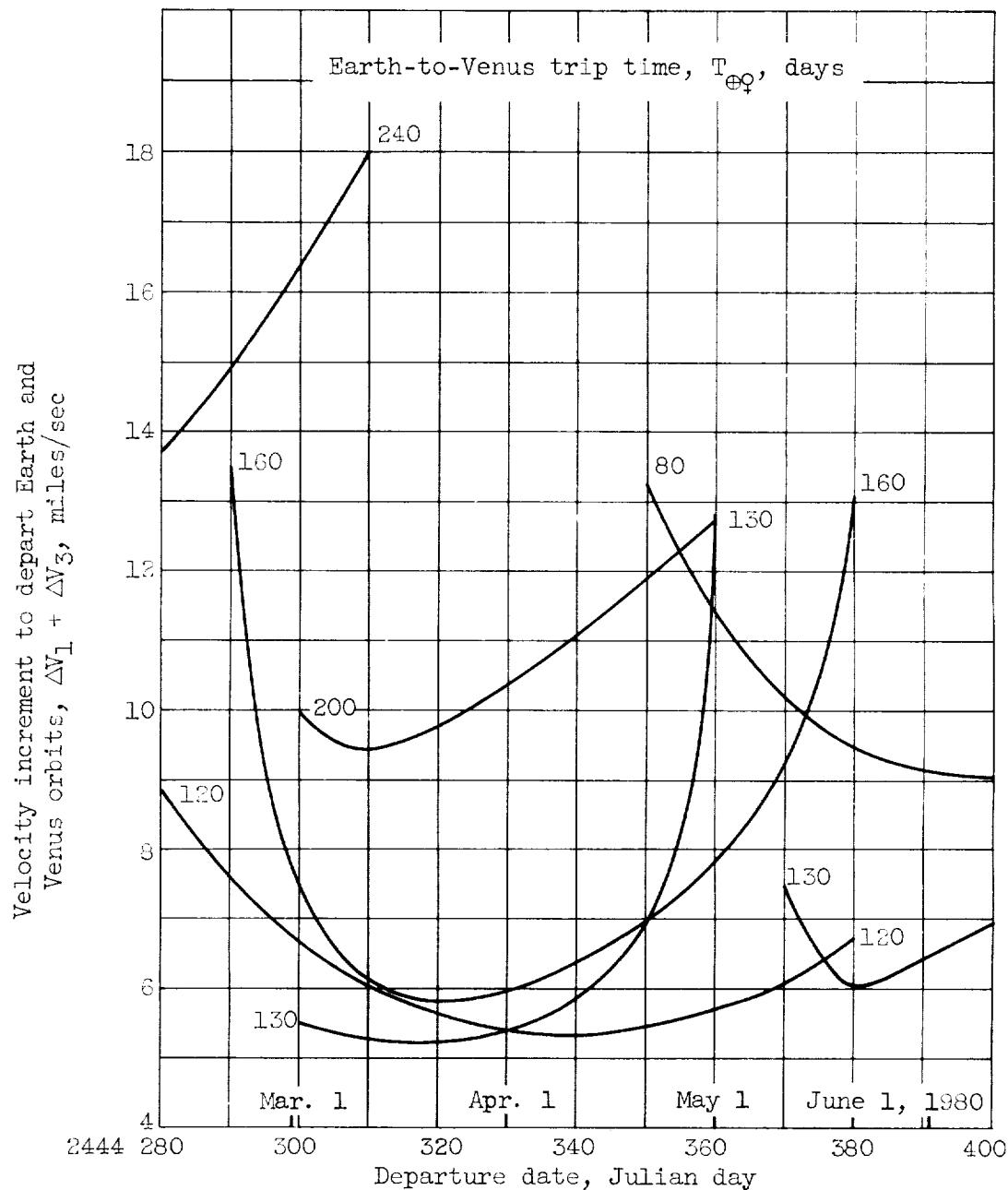
(b) Atmospheric braking at Earth.

Figure 9. - Continued. Velocity increments for 460-day round trip to Venus. Wait time in Venus orbit, 40 days.



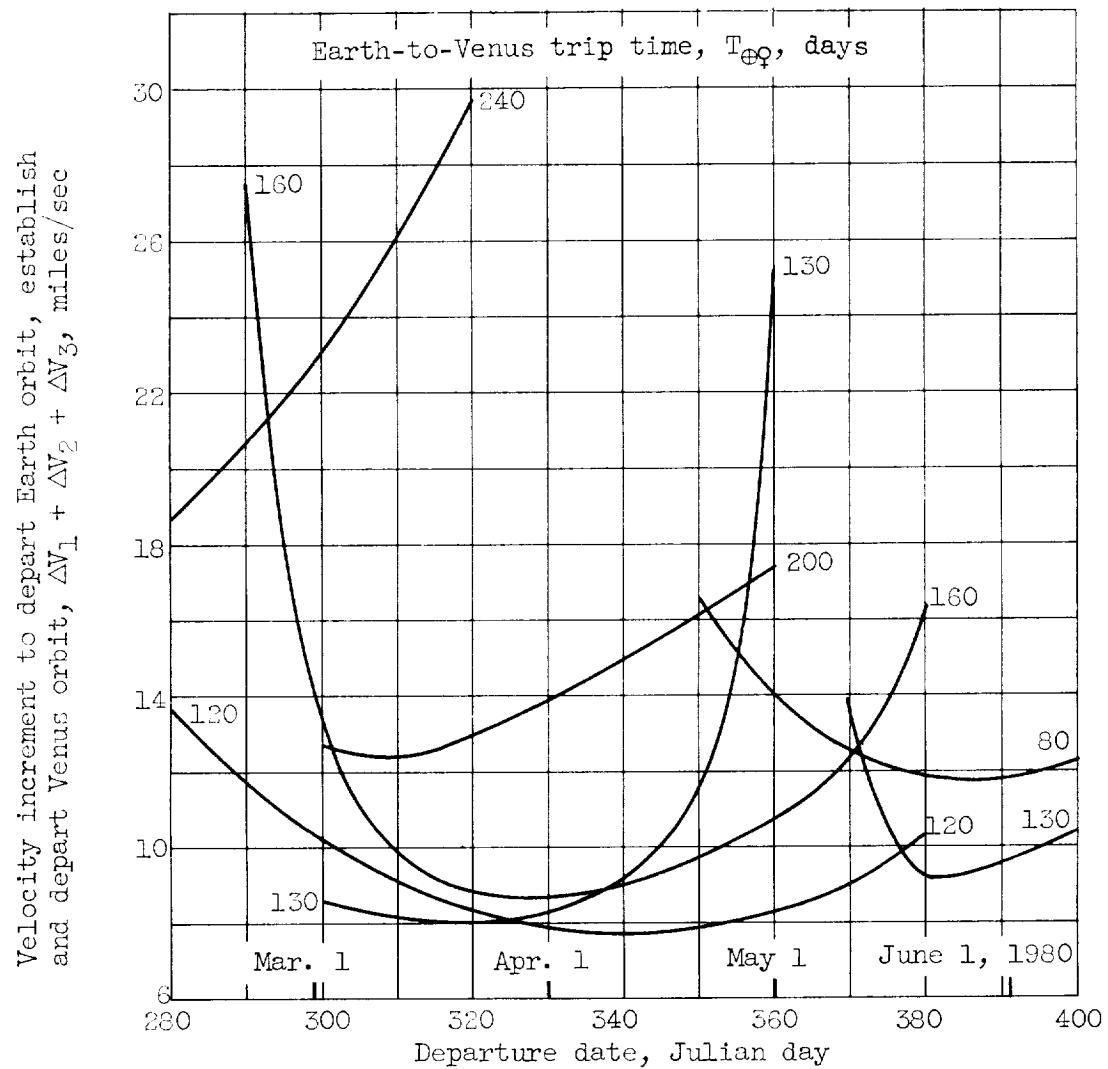
(c) All propulsive braking.

Figure 9. - Concluded. Velocity increments for 460-day round trip to Venus. Wait time in Venus orbit, 40 days.



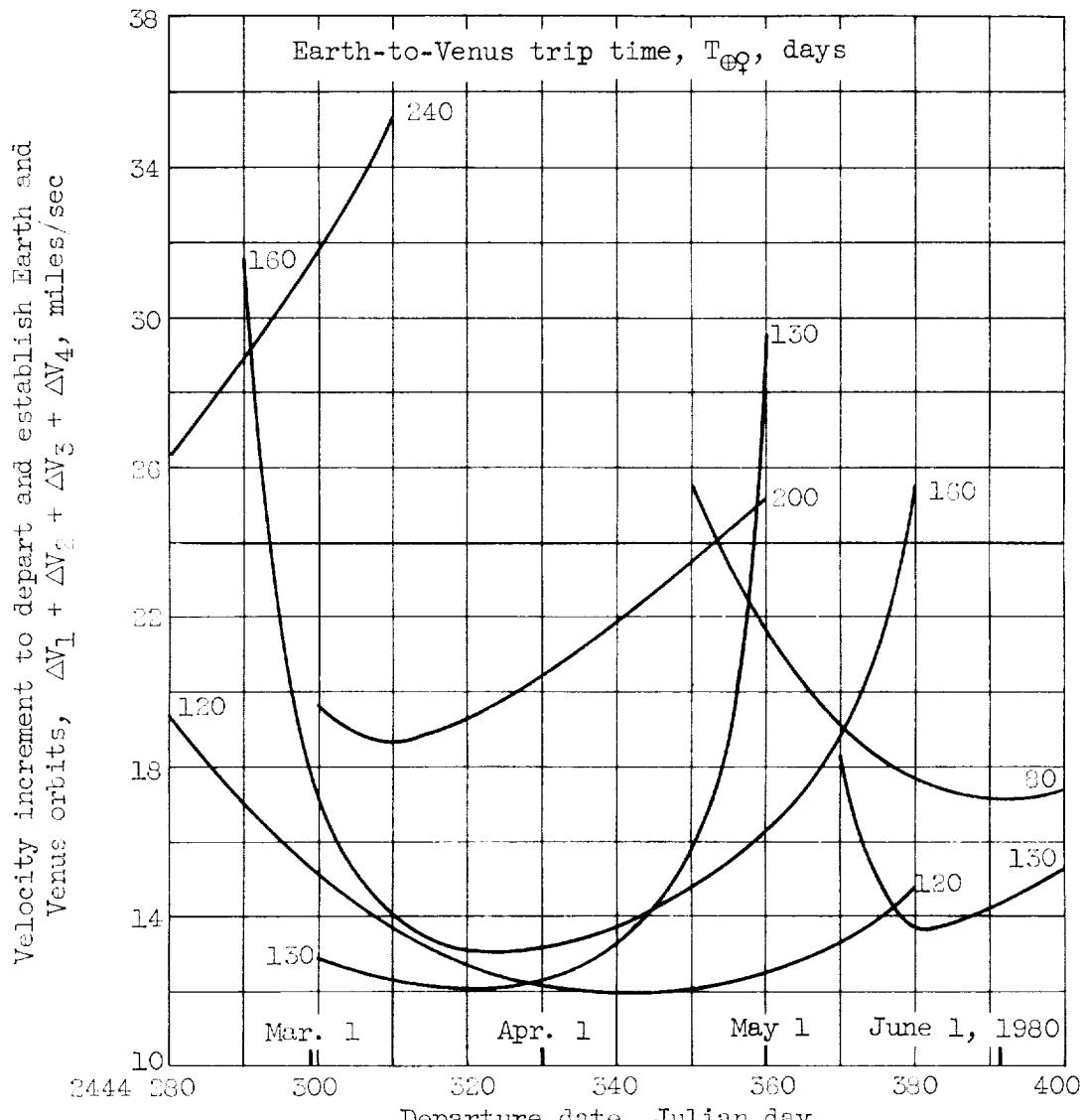
(a) Atmospheric braking at Venus and Earth.

Figure 10. - Velocity increments for 500-day round trip to Venus. Wait time in Venus orbit, 40 days.



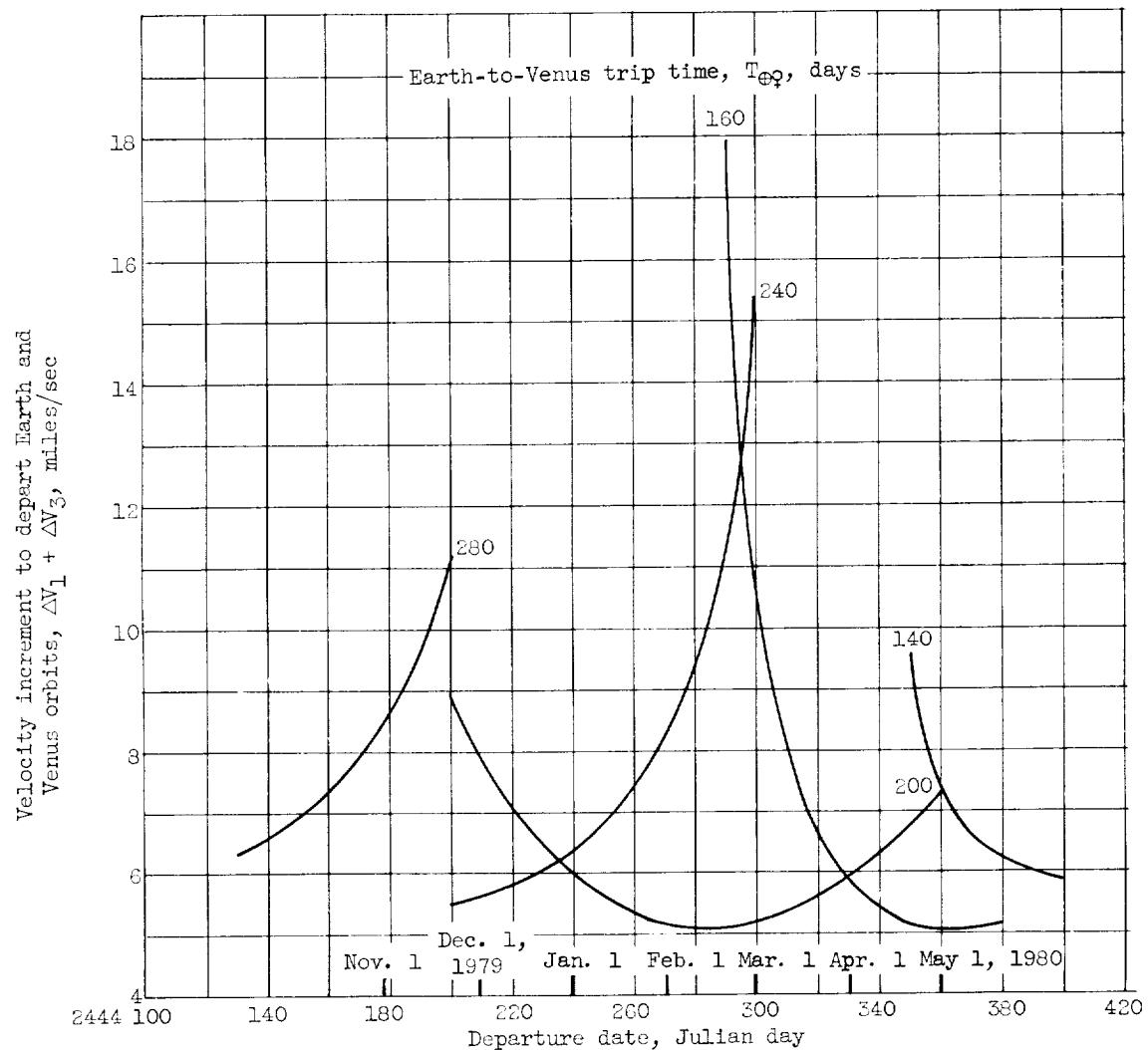
(b) Atmospheric braking at Earth.

Figure 10. - Continued. Velocity increments for 500-day round trip to Venus. Wait time in Venus orbit, 40 days.



(c) All propulsive braking.

Figure 10. - Concluded. Velocity increments for 500-day round trip to Venus. Wait time in Venus orbit, 40 days.



(a) Atmospheric braking at Venus and Earth.

Figure 11. - Velocity increments for 580-day round trip to Venus orbit,
40 days.

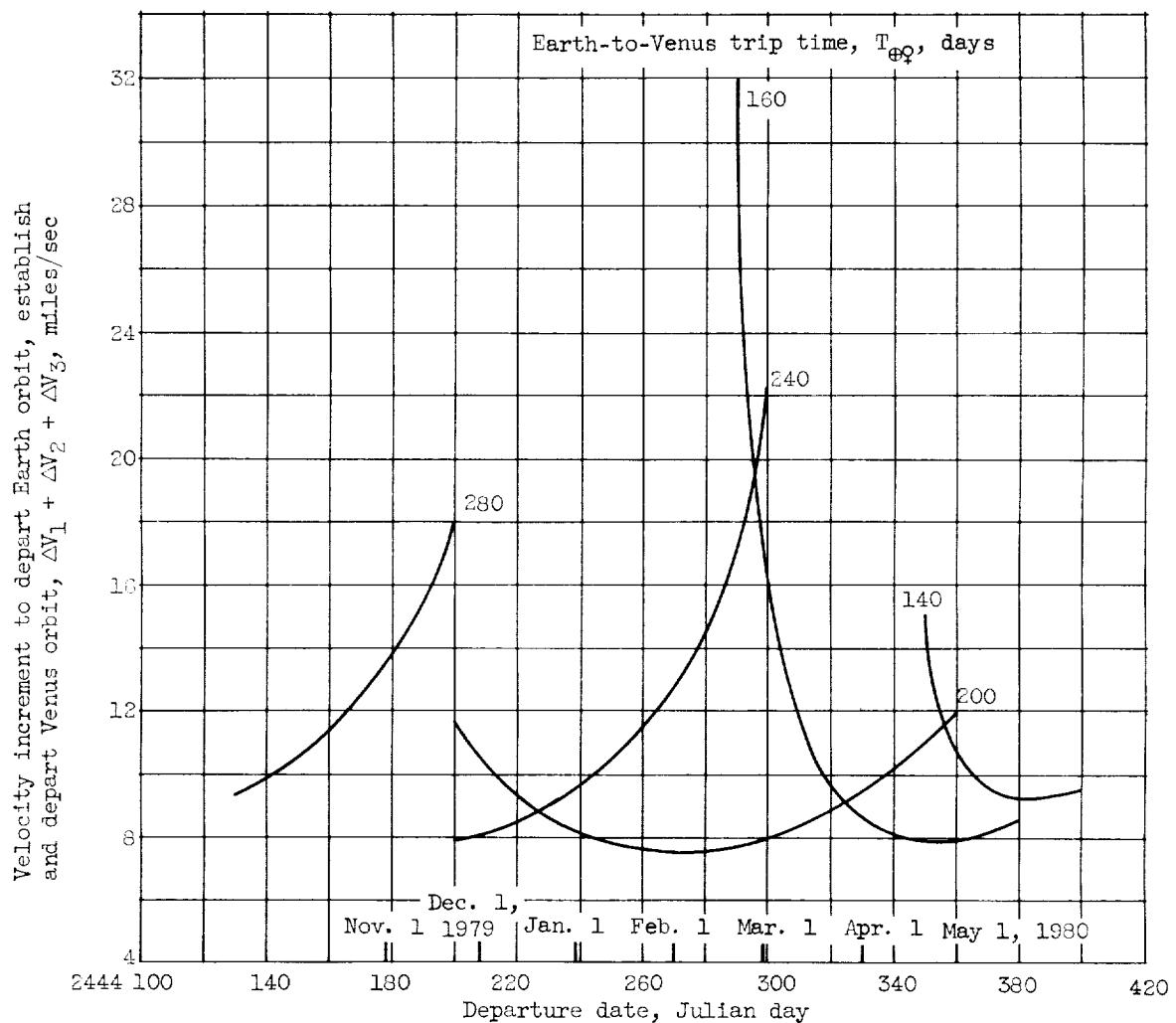
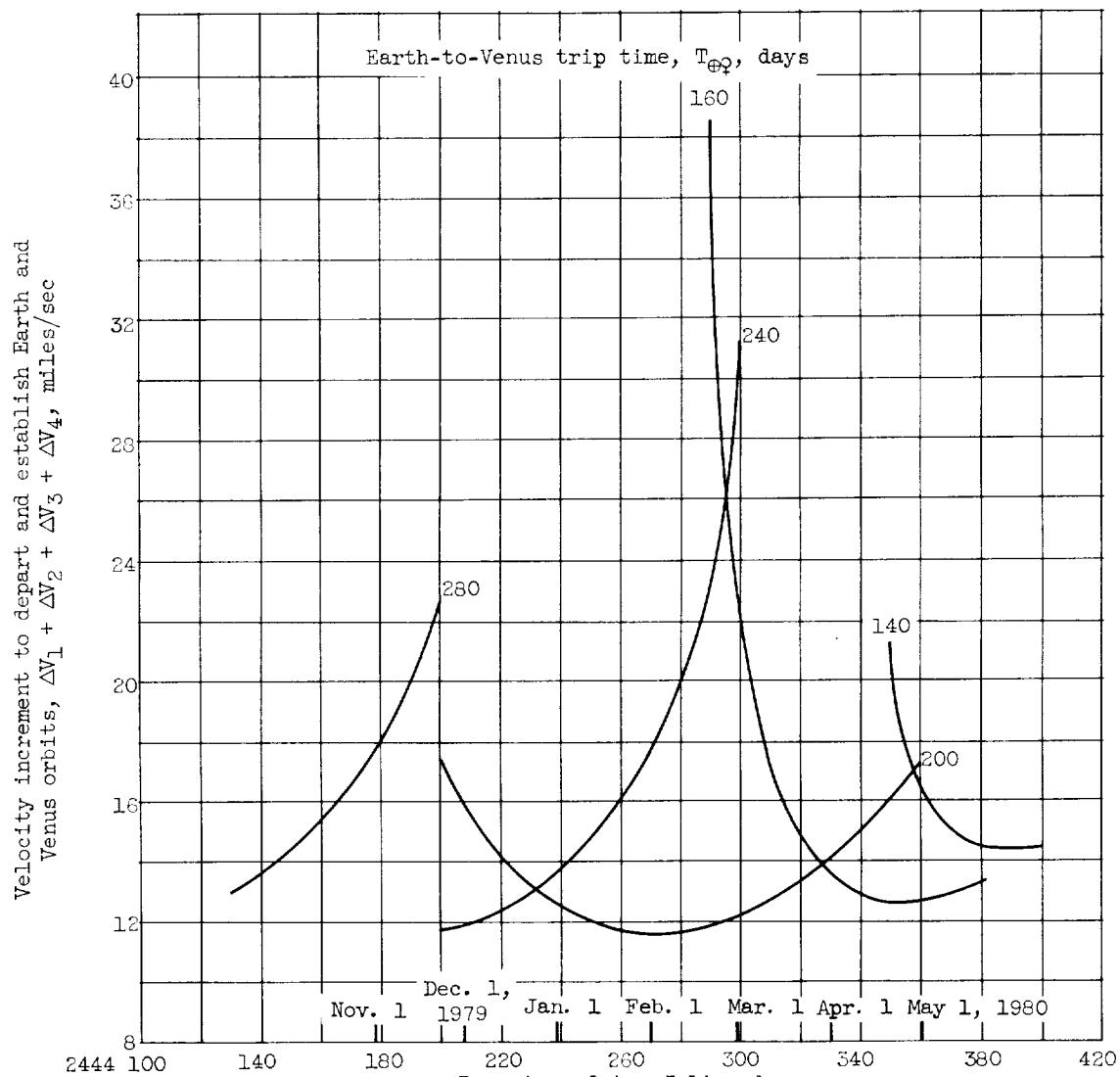
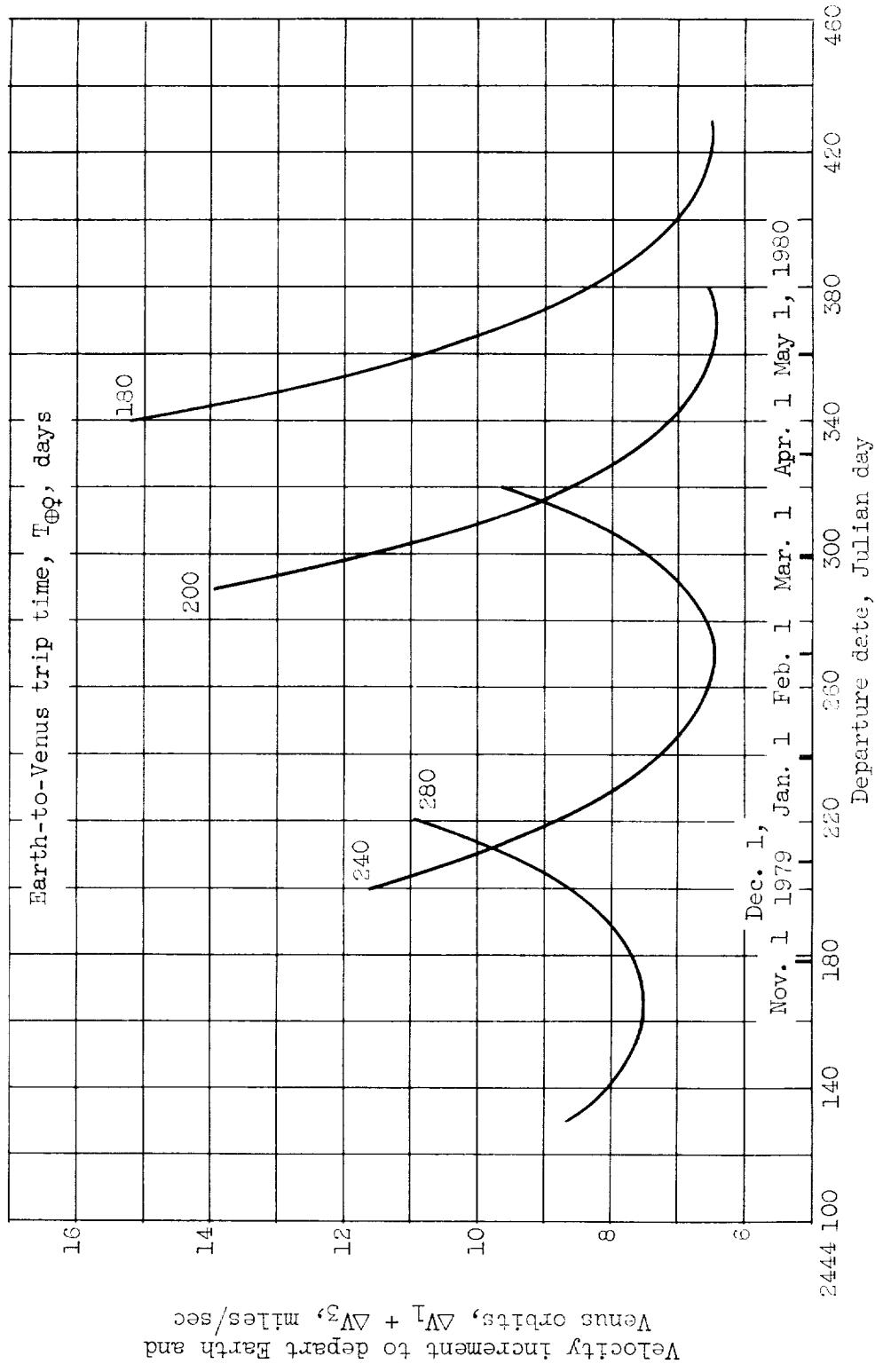


Figure 11. - Continued. Velocity increments for 580-day round trip to Venus.
Wait time in Venus orbit, 40 days.



(c) All propulsive braking.

Figure 11. - Concluded. Velocity increments for 580-day round trip to Venus.
Wait time in Venus orbit, 40 days.



(a) Atmospheric braking at Venus and Earth.

Figure 12. - Velocity increments for 660-day round trip to Venus. Wait time in Venus orbit, 40 days.

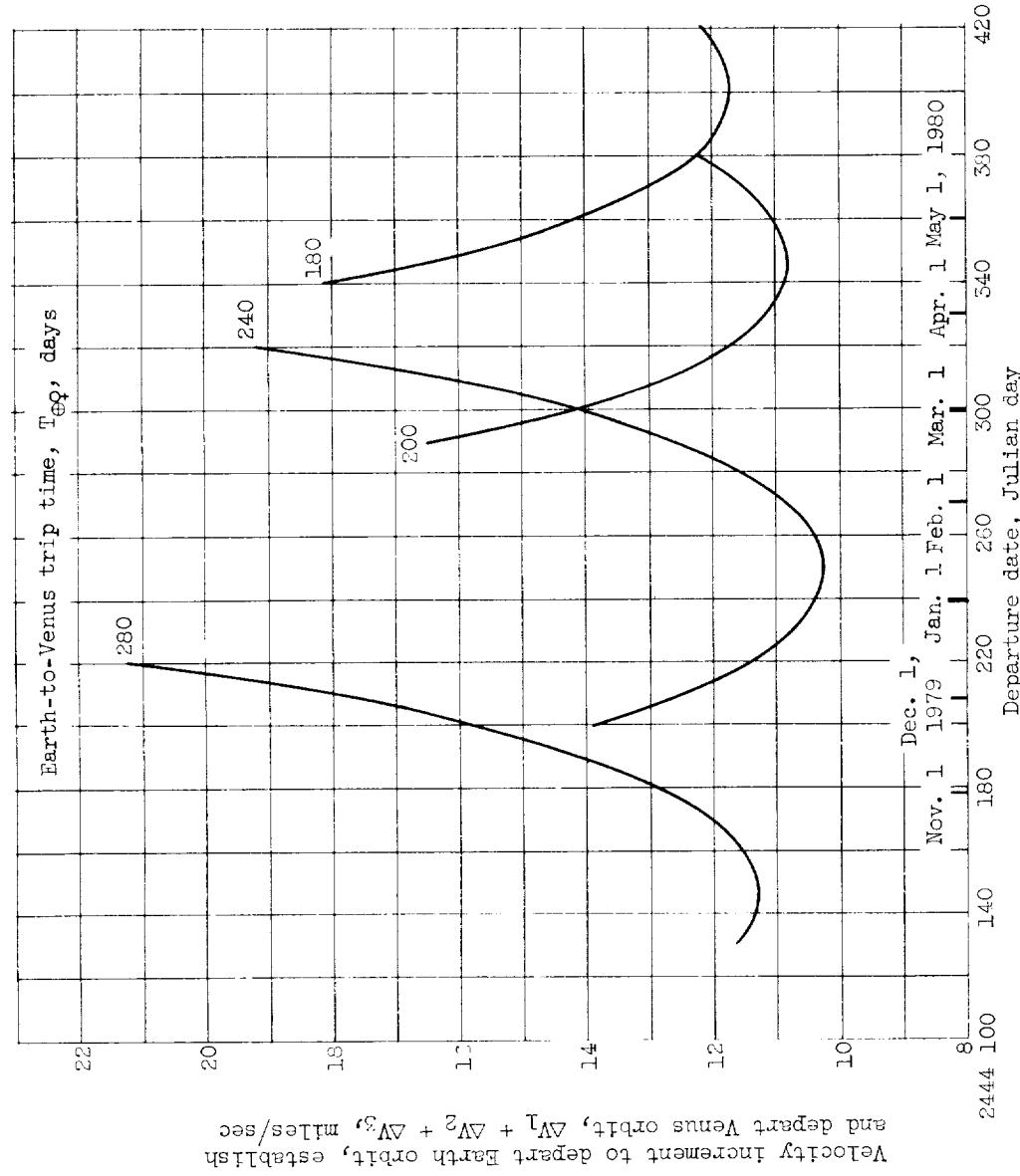
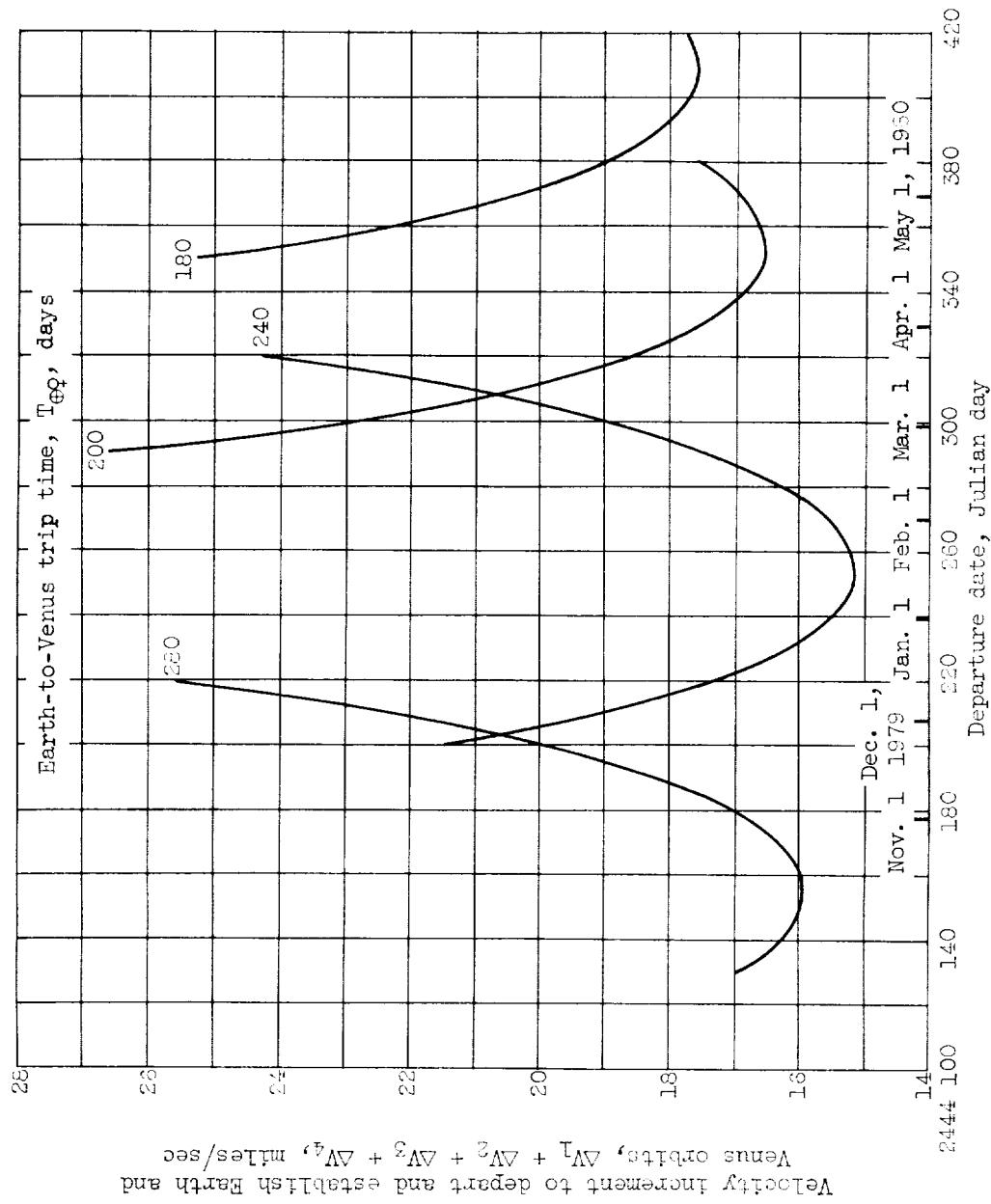
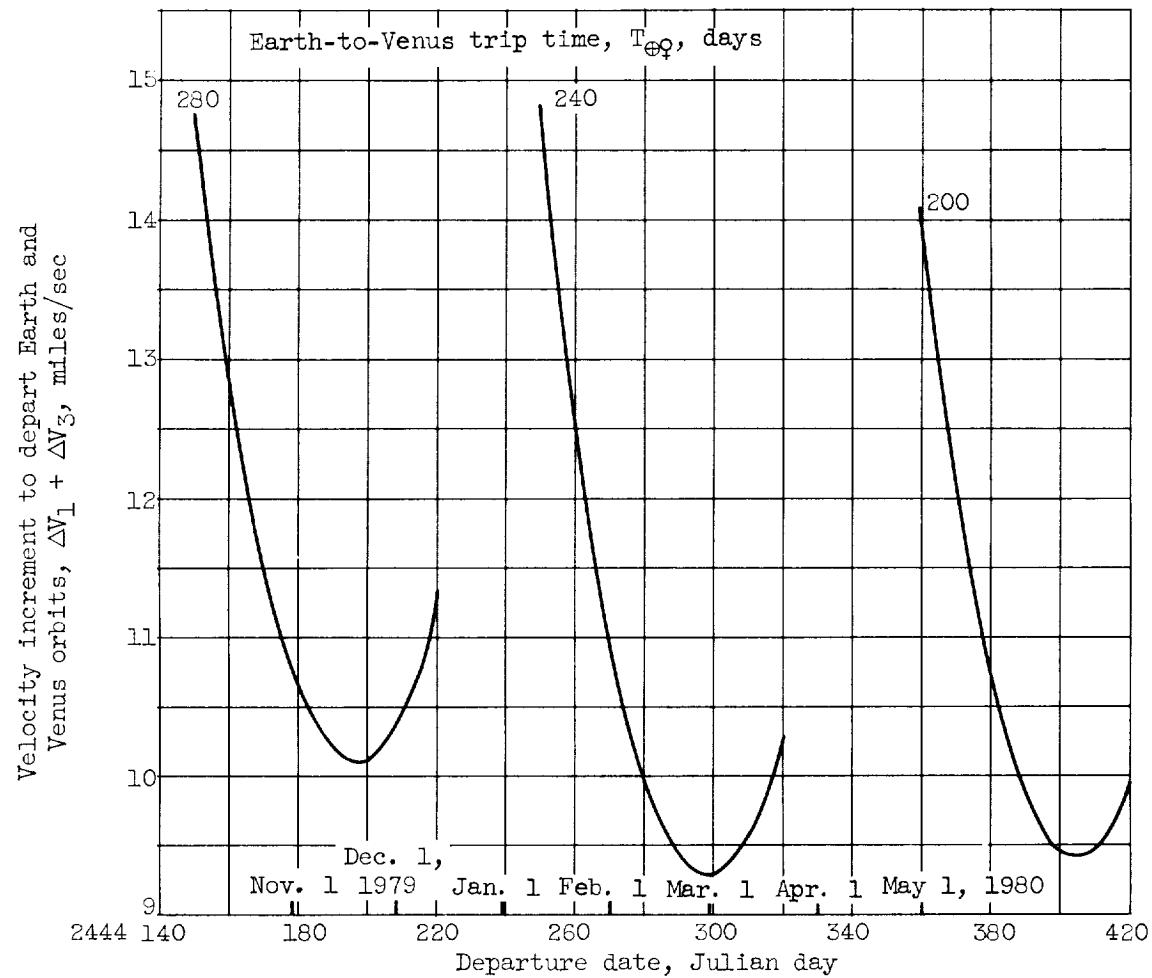


Figure 12. - Continued. Velocity increments for 660-day round trip to Venus.
Wait time in Venus orbit, 40 days.



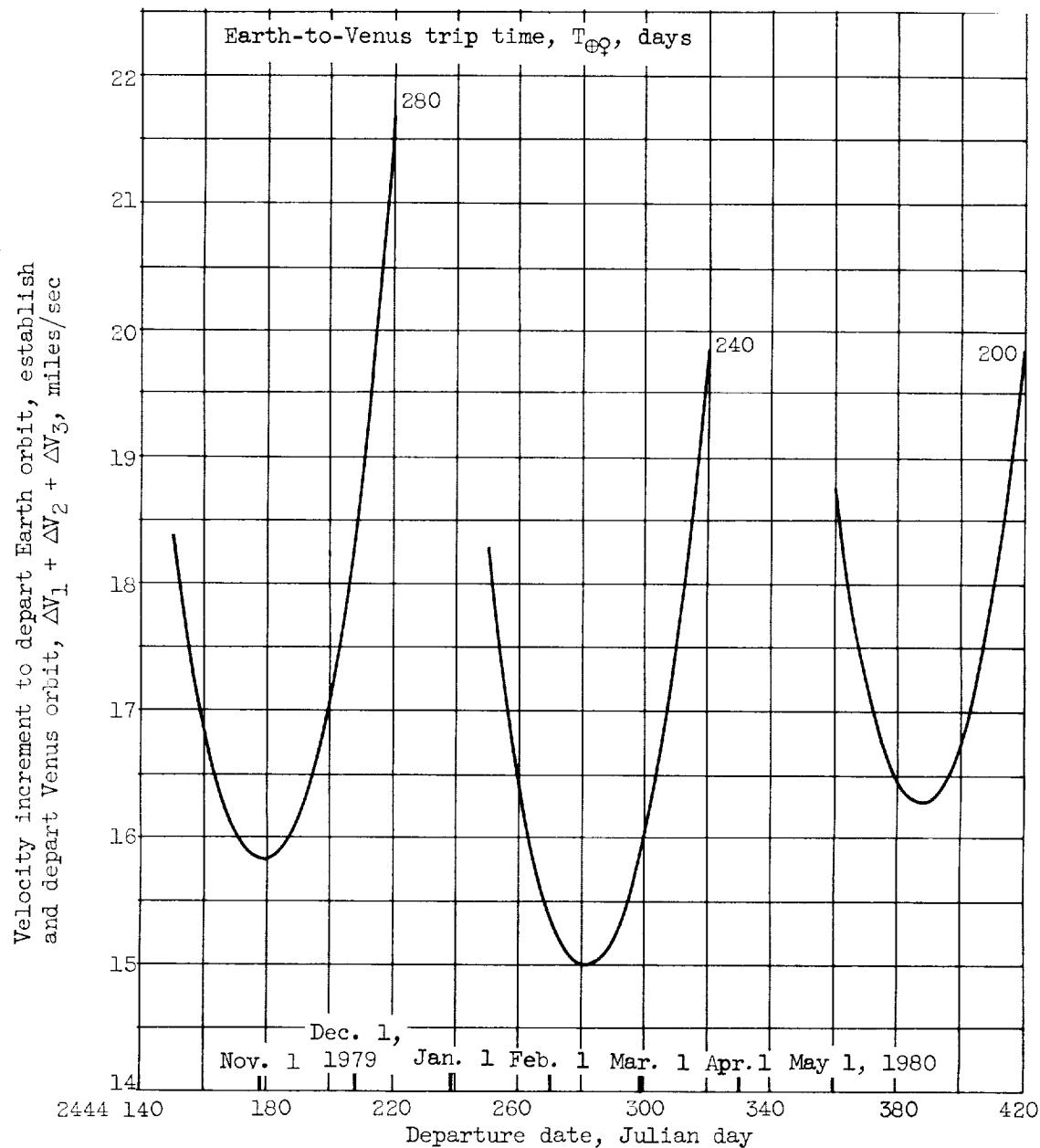
(c) All propulsive braking.

Figure 12. - Concluded. Velocity increments for 660-day round trip to Venus.
Wait time in Venus orbit, 40 days.



(a) Atmospheric braking at Venus and Earth.

Figure 13. - Velocity increments for 700-day round trip to Venus. Wait time in Venus orbit, 40 days.



(b) Atmospheric braking at Earth.

Figure 13. - Continued. Velocity increments for 700-day round trip to Venus. Wait time in Venus orbit, 40 days.

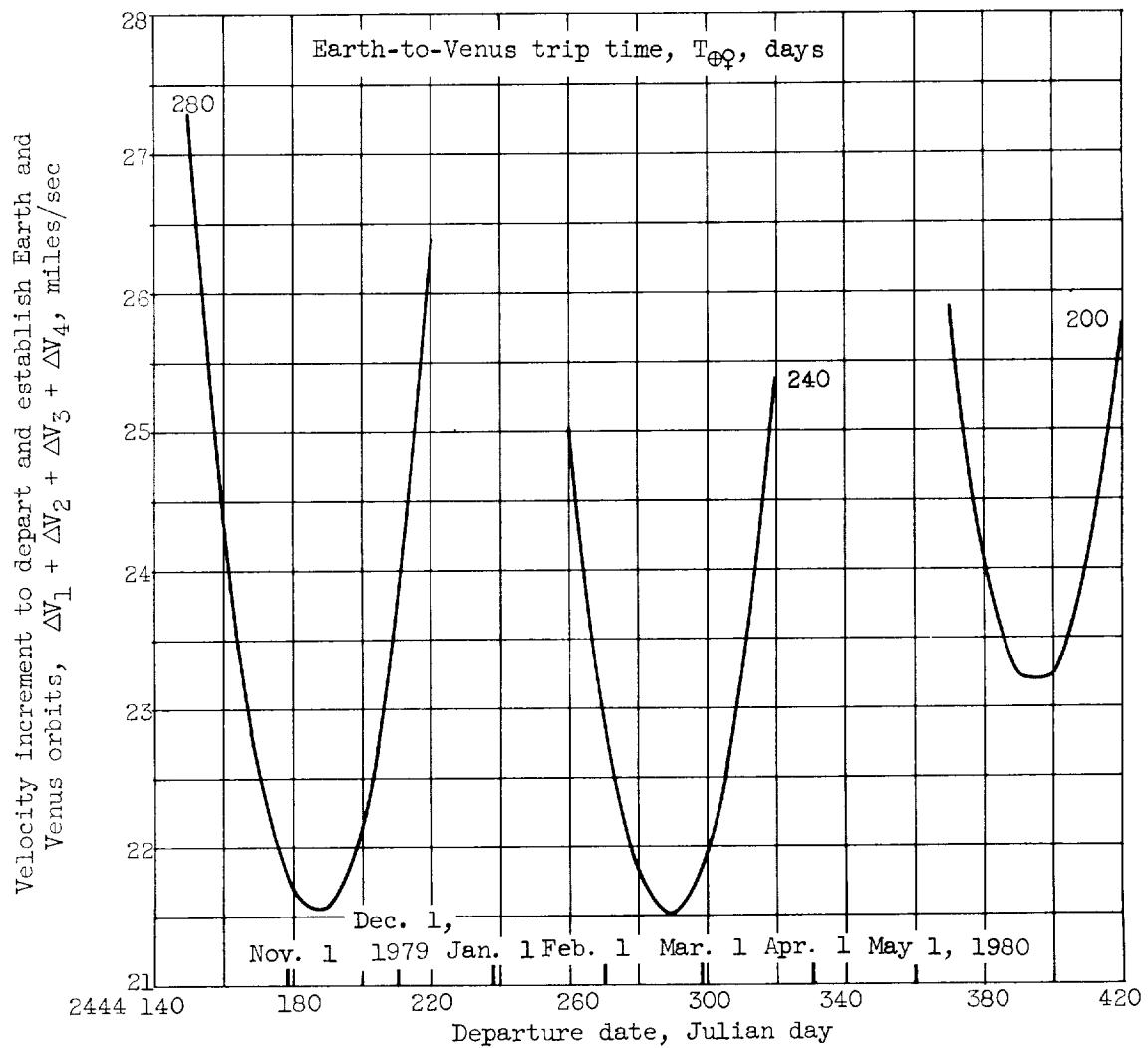


Figure 13. - Concluded. Velocity increments for 700-day round trip to Venus. Wait time in Venus orbit, 40 days.

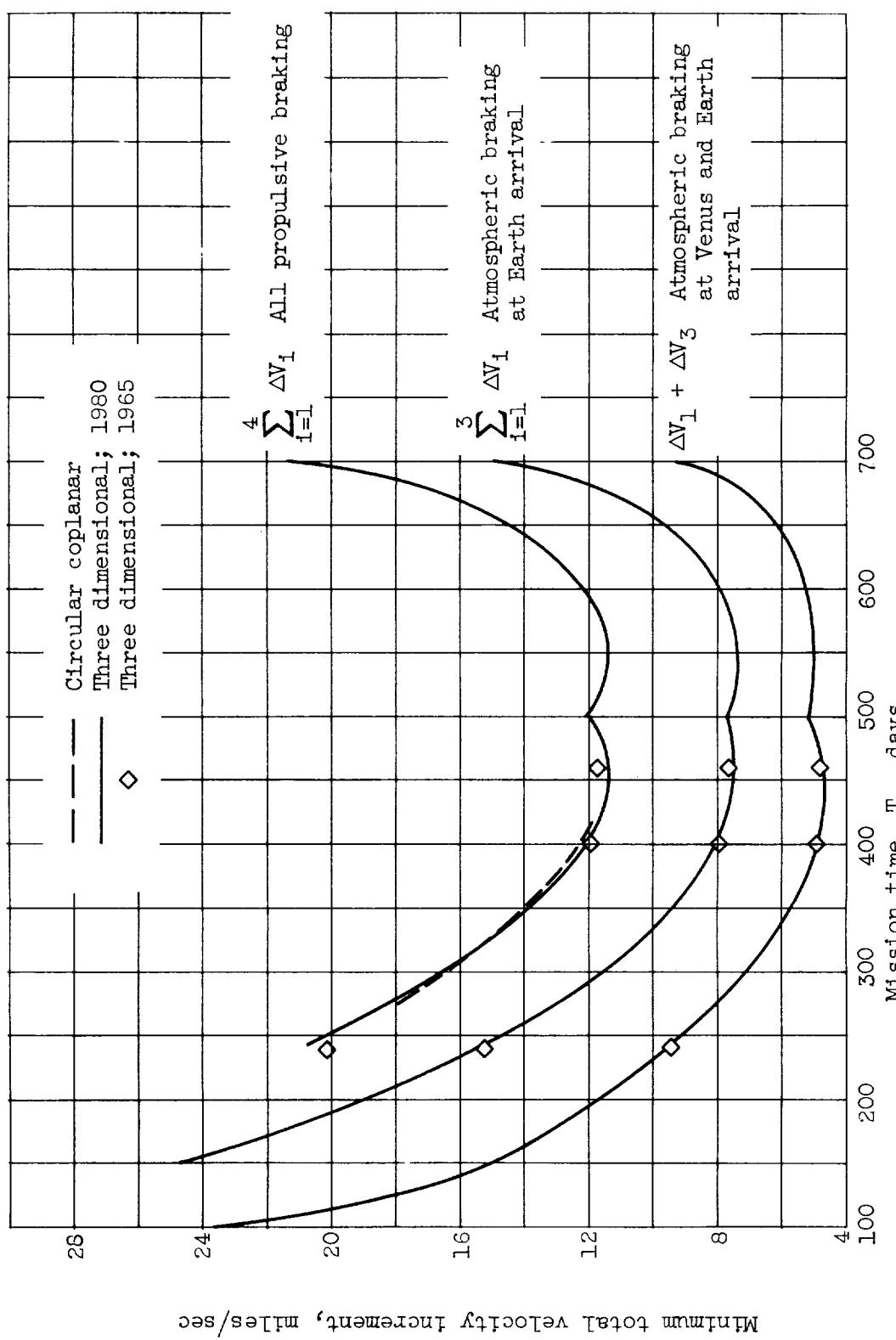
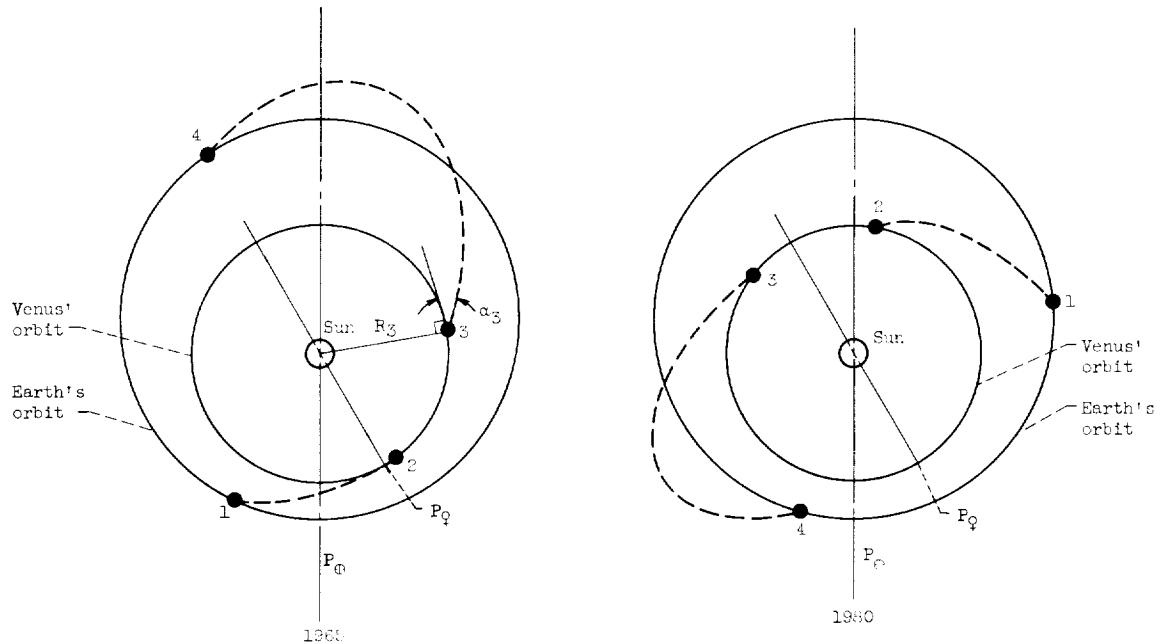


Figure 14. - Effect of atmospheric braking on total velocity increment requirements for round trips to Venus. Wait time in Venus orbit, 40 days.



Trajectory parameter	1961	1980
	Route 1-2	
Heliocentric radius of vehicle at Earth departure, R ₁ , miles	91.83x10 ⁶	93.48x10 ⁶
Heliocentric radius of vehicle at Venus arrival, R ₂ , miles	66.74x10 ⁶	67.58x10 ⁶
Heliocentric velocity to depart Earth orbit, V ₁ , miles/sec	15.78	15.41
Heliocentric velocity on arrival at Venus orbit, V ₂ , miles/sec	22.1	22.31
Heliocentric path of vehicle on Earth departure, α ₁ , deg	-7.63	-8.16
Heliocentric path of vehicle on Venus arrival, α ₂ , deg	-17.70	-18.03
Velocity of Earth on departure, V _{E1} , miles/sec	16.77	18.38
Velocity of Venus on arrival, V _{Q2} , miles/sec	21.89	21.62
Route 3-4		
Heliocentric radius of vehicle at Venus departure, R ₃ , miles	67.08x10 ⁶	67.19x10 ⁶
Heliocentric radius of vehicle at Earth arrival, R ₄ , miles	94.27x10 ⁶	91.38x10 ⁶
Heliocentric velocity to depart Venus orbit, V ₃ , miles/sec	22.68	22.43
Heliocentric velocity on arrival at Earth orbit, V ₄ , miles/sec	16.51	16.07
Heliocentric path of vehicle on Venus departure, α ₃ , deg	26.44	26.05
Heliocentric path of vehicle on Earth arrival, α ₄ , deg	-21.39	-21.36
Velocity of Venus on departure, V _{Q3} , miles/sec	21.79	21.62
Velocity of Earth on arrival, V _{E4} , miles/sec	18.22	18.80

Figure 18. - Comparison of 240-day round-trip missions to Venus in 1961 and 1980. Wait time in Venus orbit, 40 days; data for minimum total velocity increment.

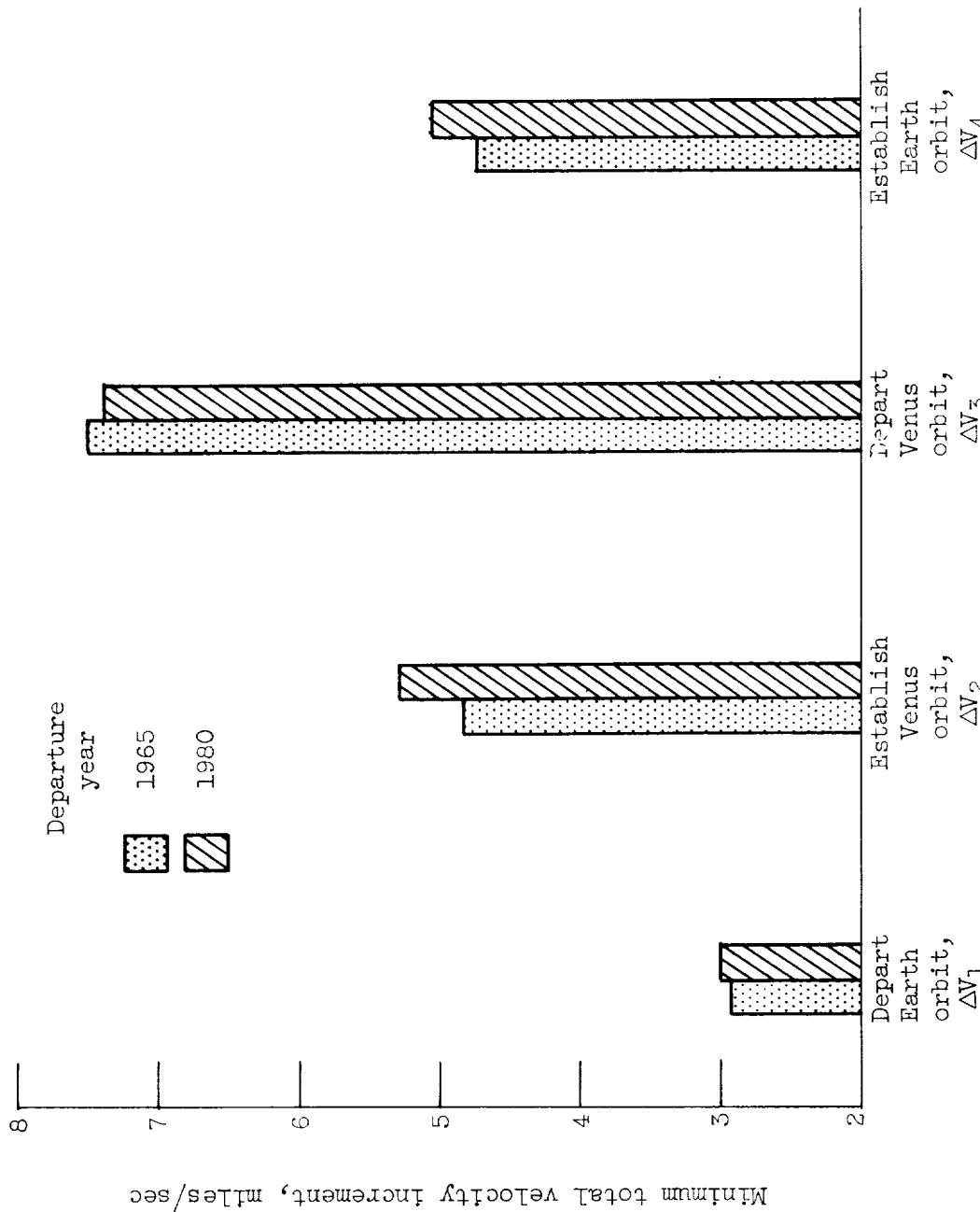
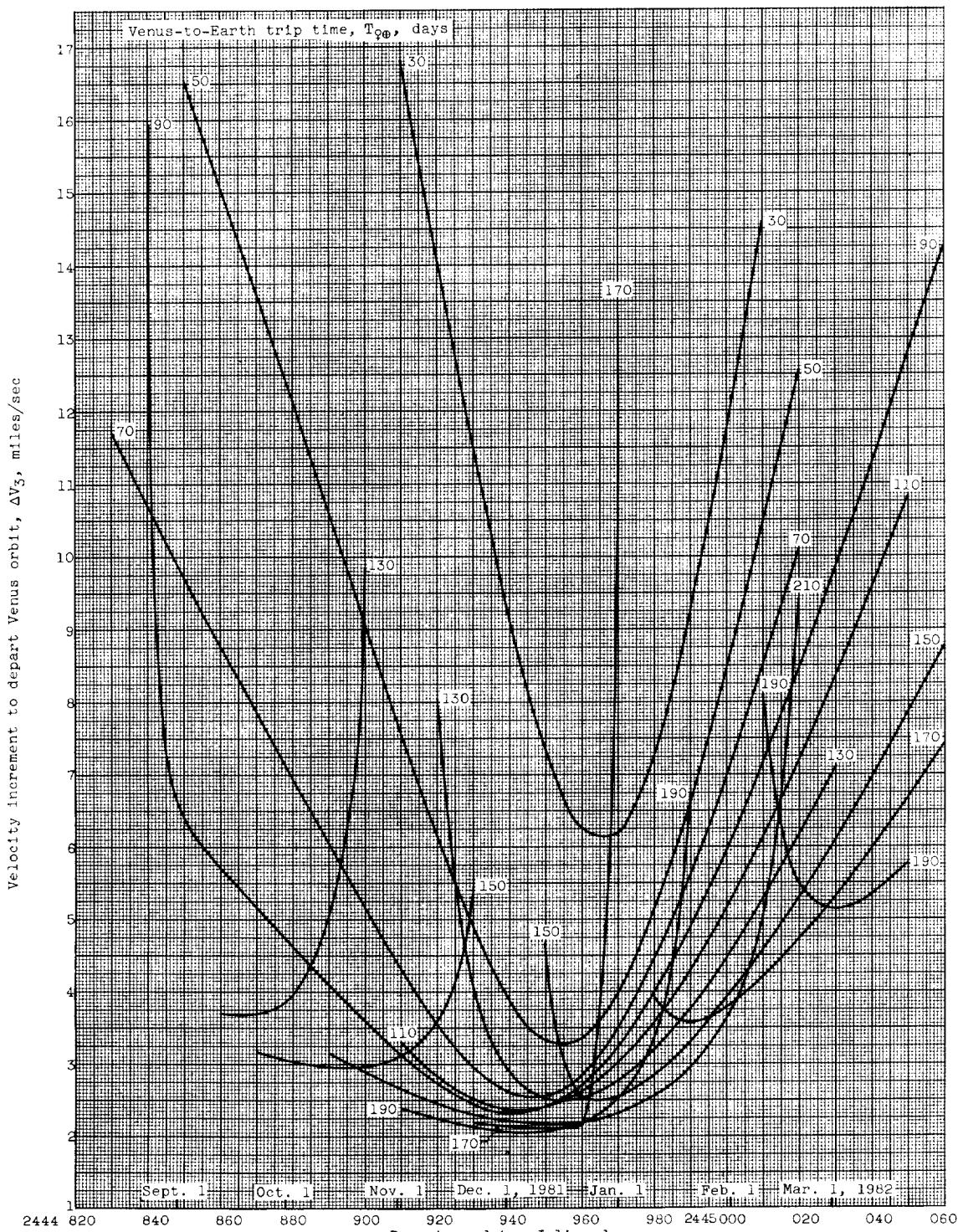
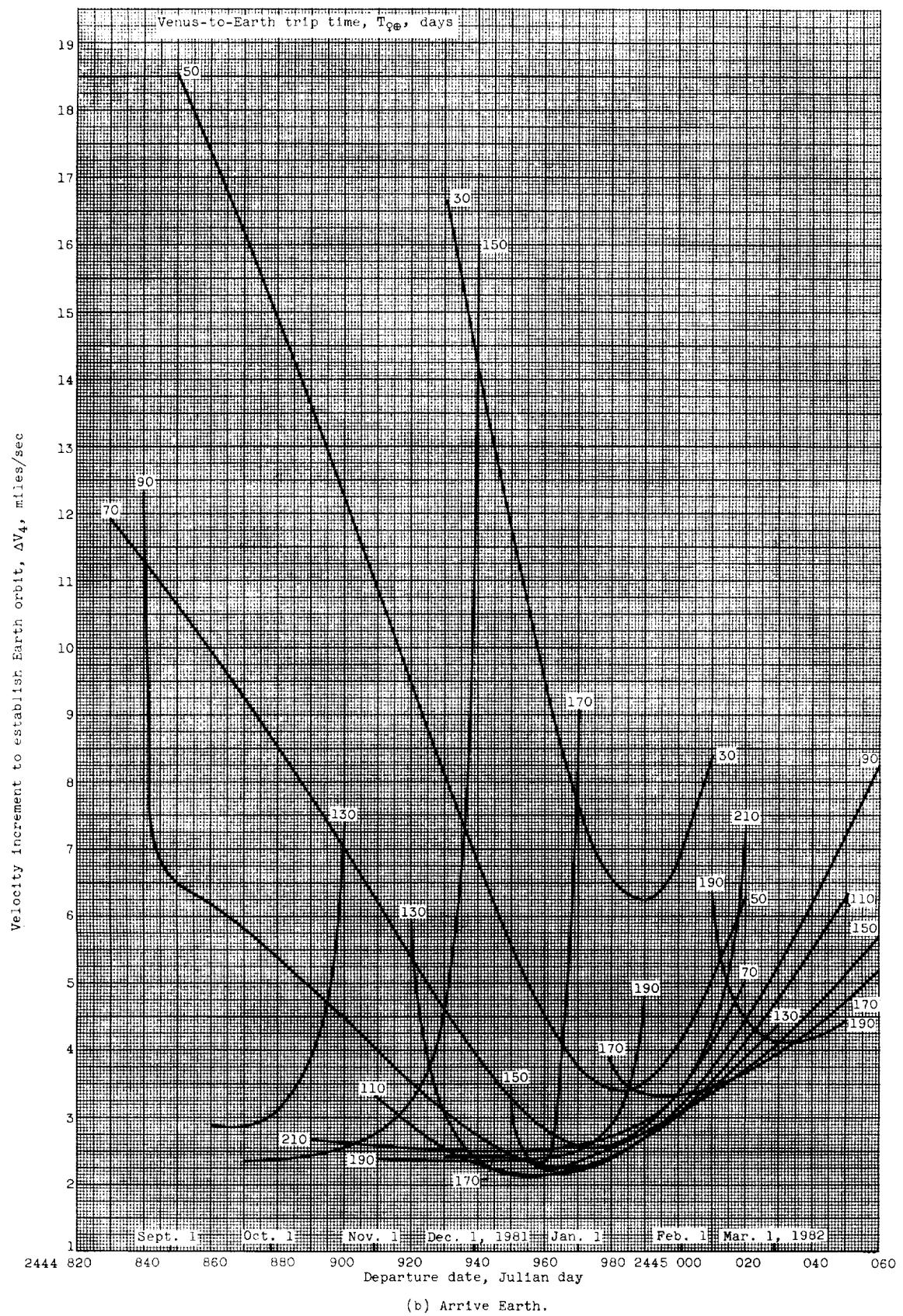


Figure 16. - Effect of synodic period of departure on velocity increments of 240-day round trip to Venus. Wait time in Venus orbit, 40 days.



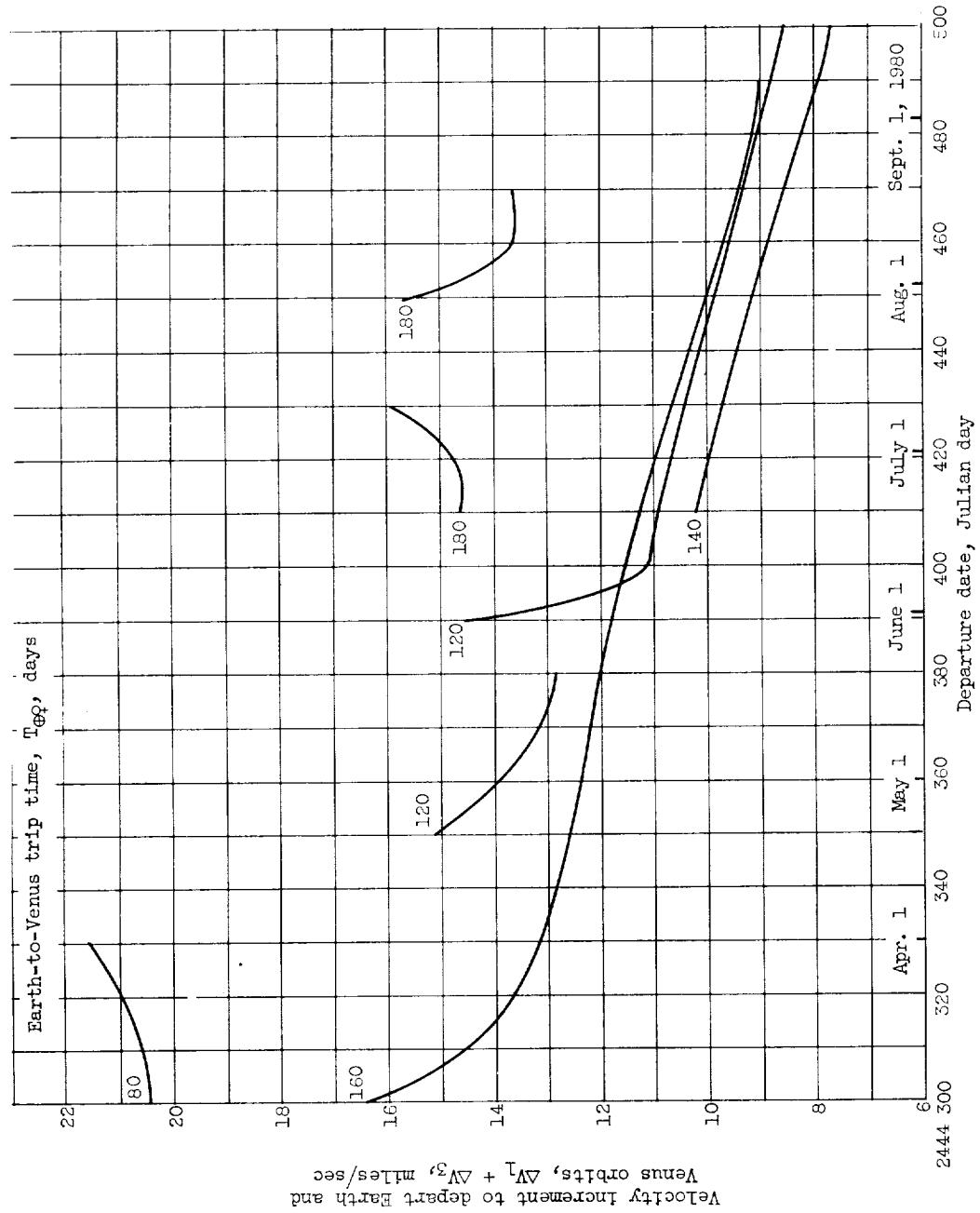
(a) Depart Venus.

Figure 17. - Velocity increments for trips from Venus to Earth starting and ending in circular orbit at 1.1 planet radii, 1981-82.



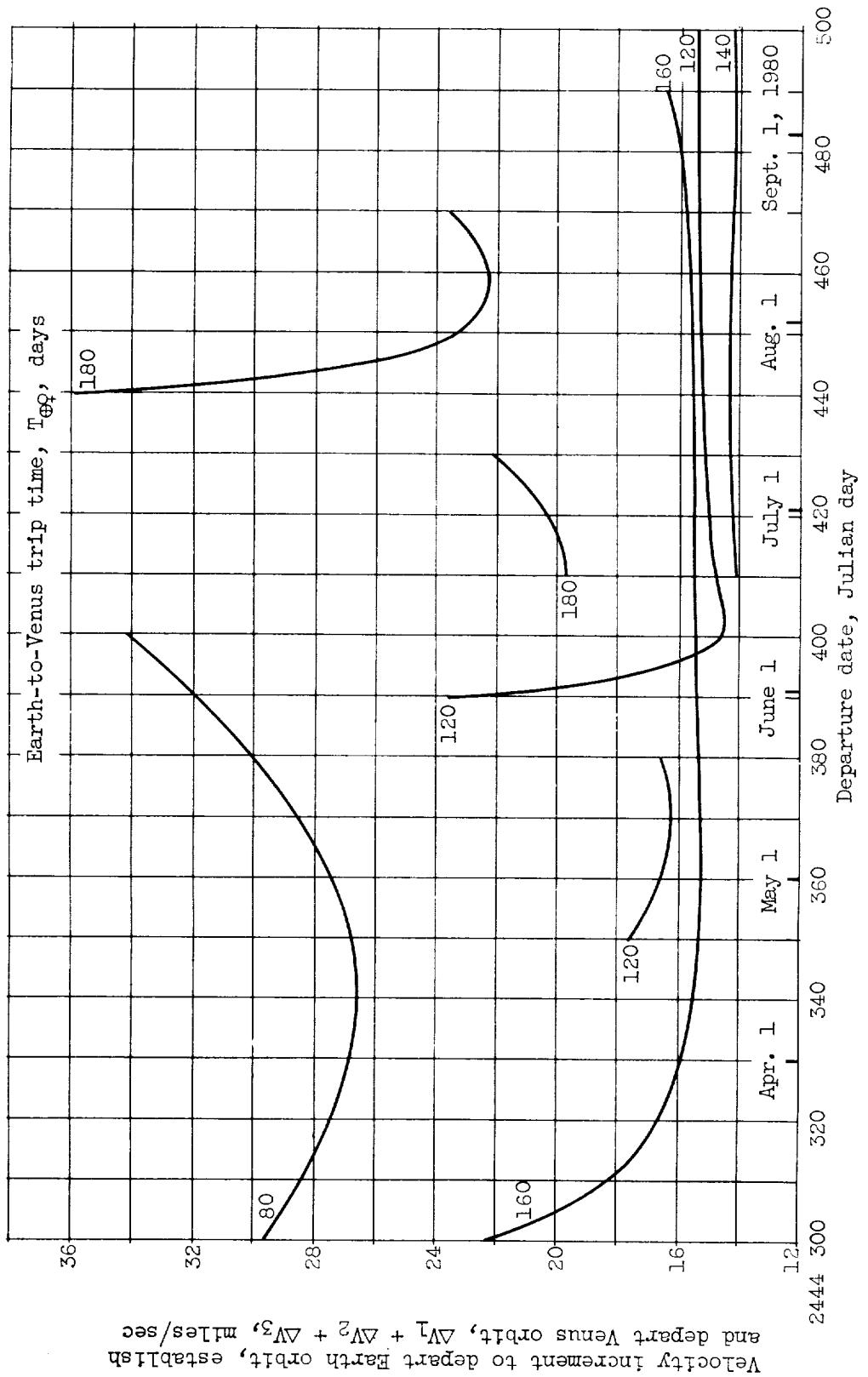
(b) Arrive Earth.

Figure 17. - Concluded. Velocity increments for trips from Venus to Earth starting and ending in circular orbit at 1.1 planet radii, 1981-82.



(a) Atmospheric braking at Venus and Earth.

Figure 18. - Velocity increments for 460-day round trip to Venus. Wait time in Venus orbit, 200 days.



(b) Atmospheric braking at Earth.

Figure 18. - Continued. Velocity increments for 460-day round trip to Venus. Wait time in Venus orbit, 200 days.

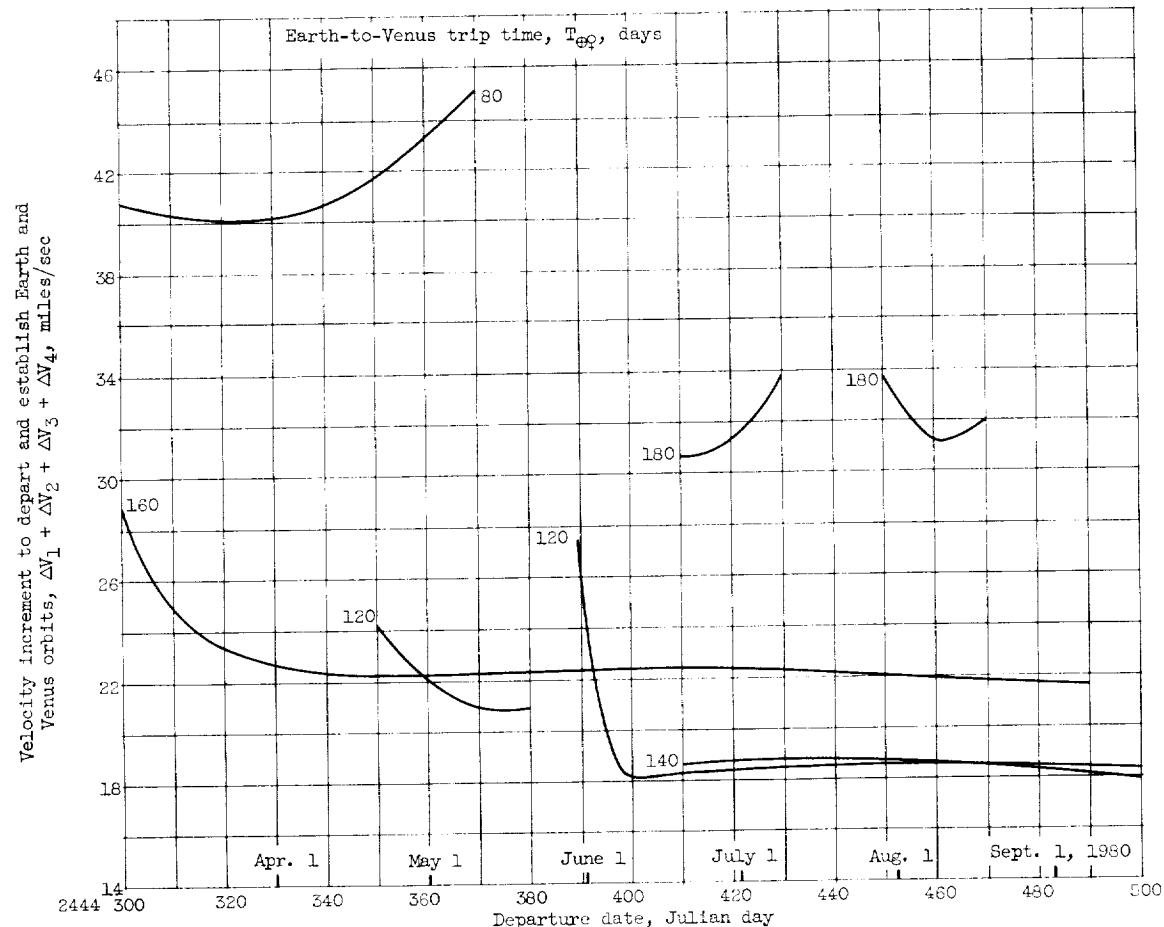
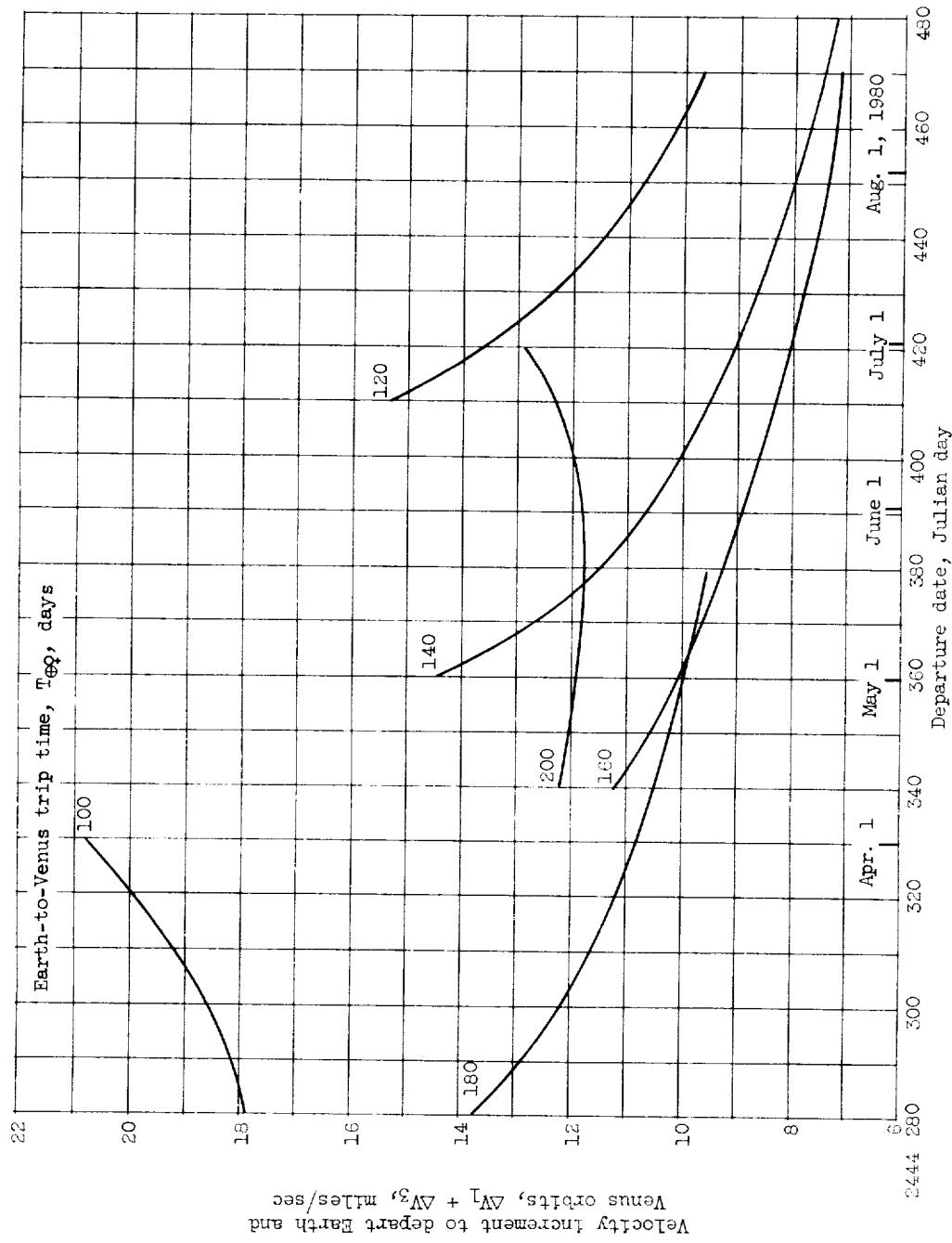
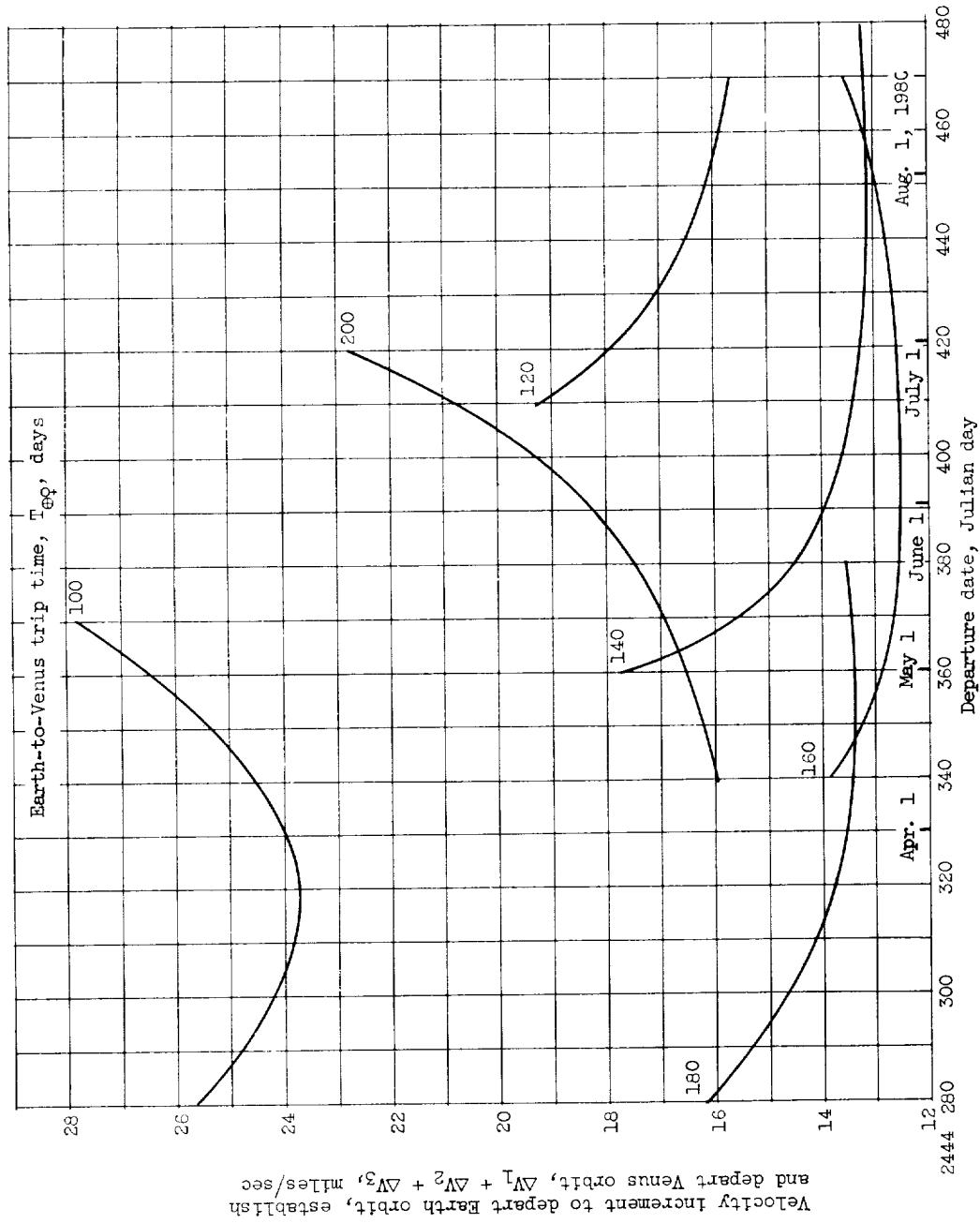


Figure 18. - Concluded. Velocity increments for 460-day round trip to Venus. Wait time in Venus orbit, 200 days.



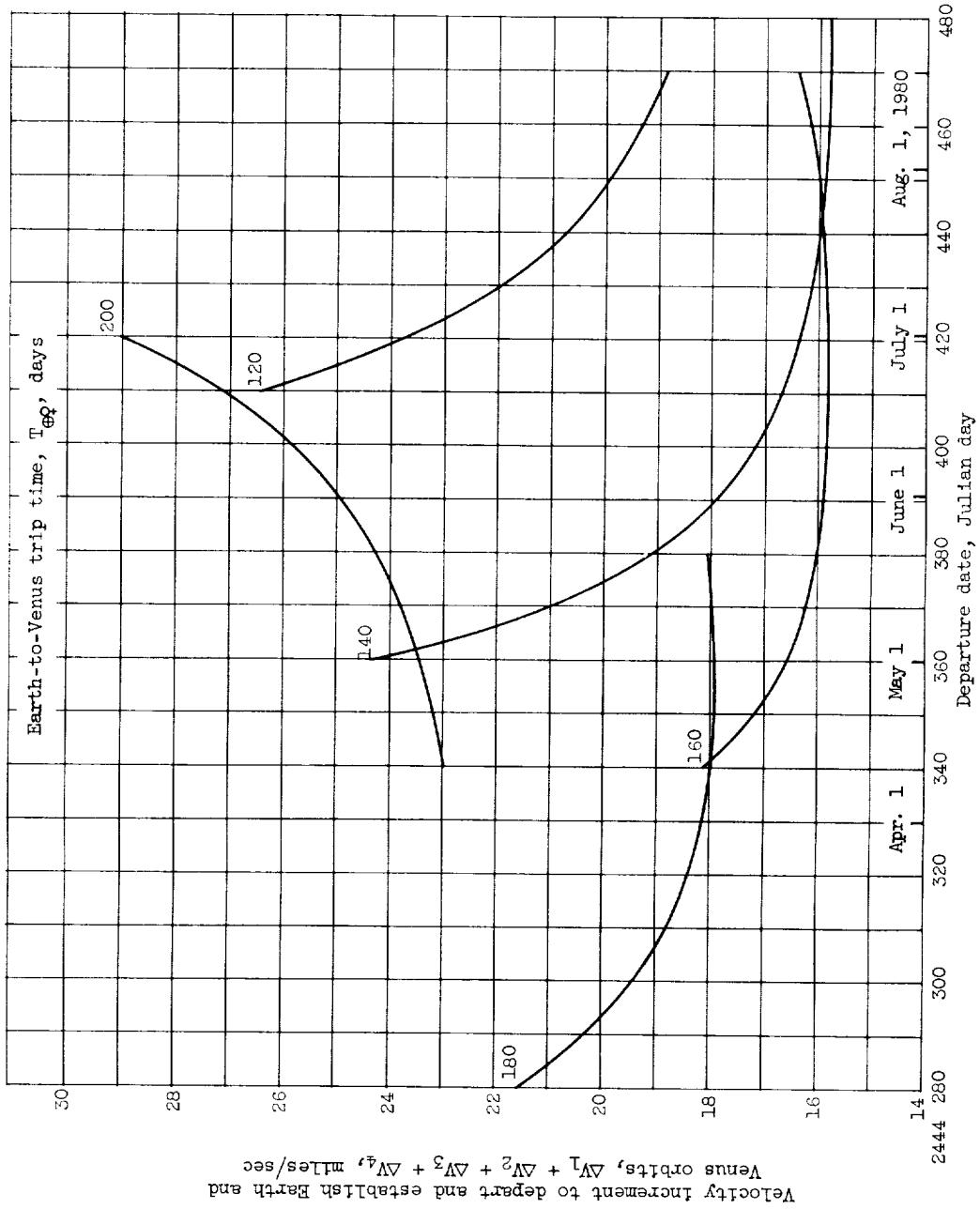
(a) Atmospheric braking at Venus and Earth.

Figure 19. - Velocity increments for 500-day round trip to Venus. Wait time in Venus orbit, 200 days.



(b) Atmospheric braking at Earth.

Figure 19. - Continued. Velocity increments for 500-day round trip to Venus. Wait time in Venus orbit, 200 days.



(c) All propulsive braking.

Figure 19. - Concluded. Velocity increments for 200-day round trip to Venus. Wait time in Venus orbit, 200 days.

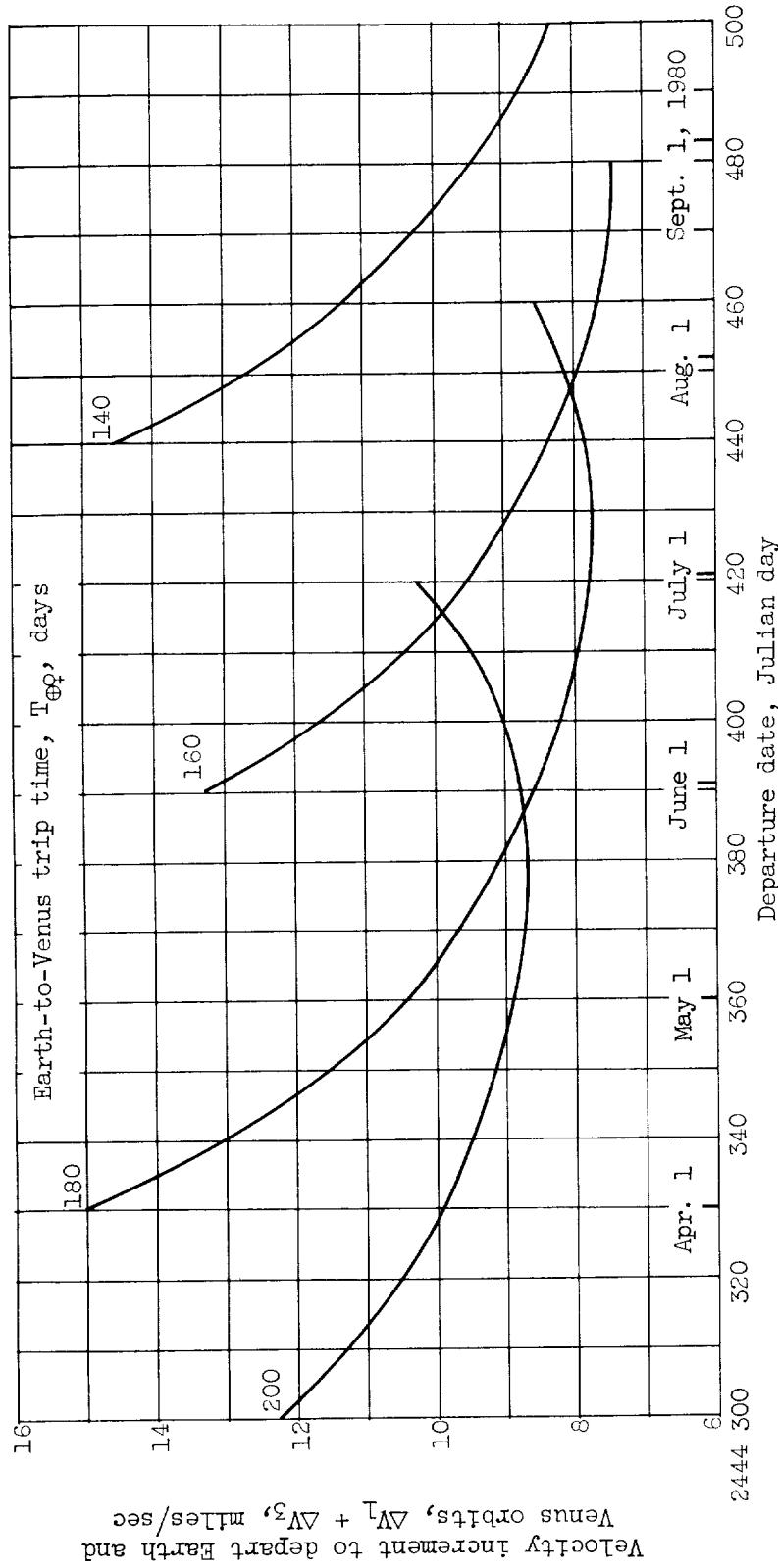


Figure 20. - Velocity increments for 550-day round trip to Venus. Wait time in Venus orbit, 200 days.

(a) Atmospheric braking at Venus and Earth.

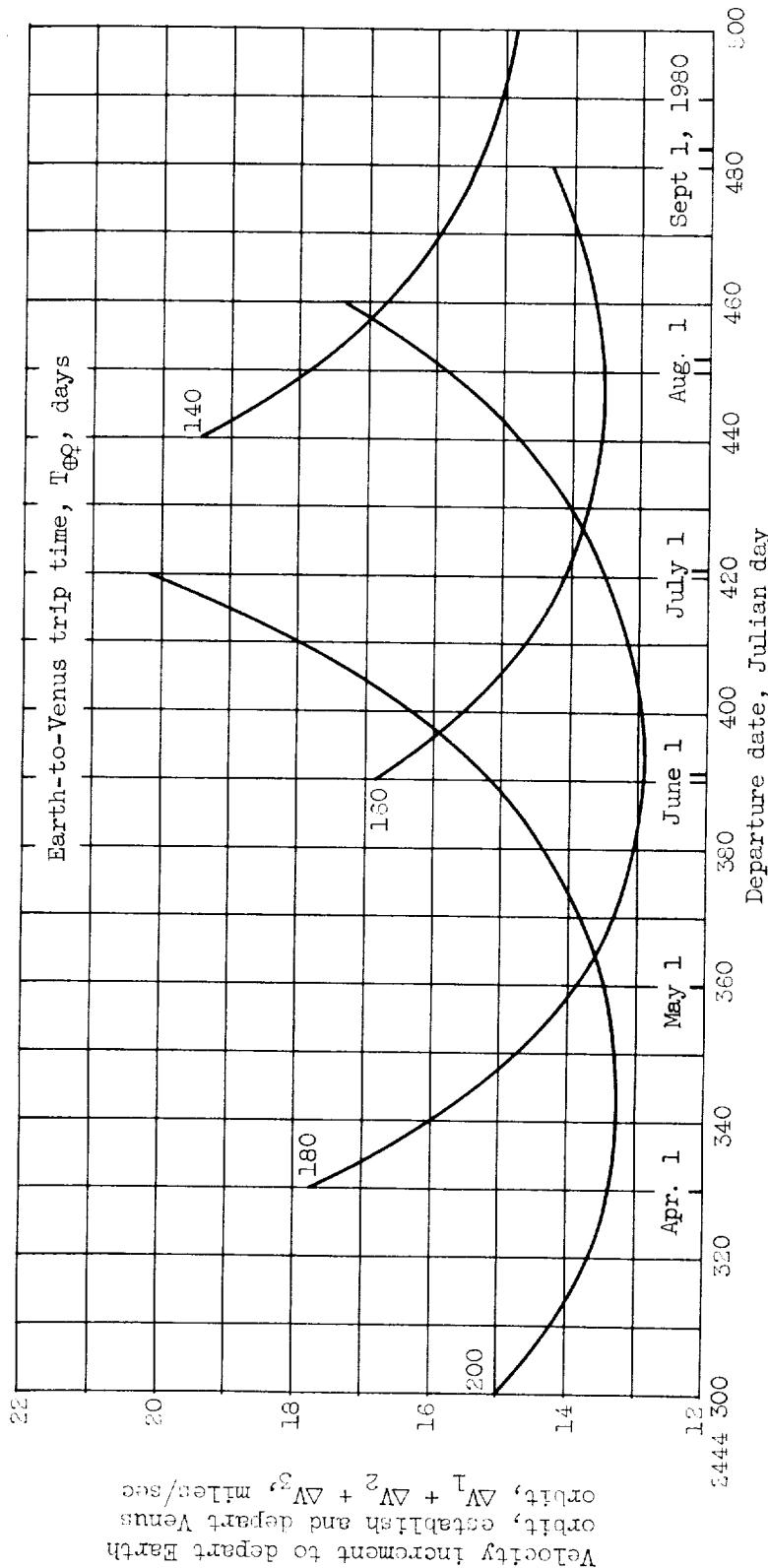
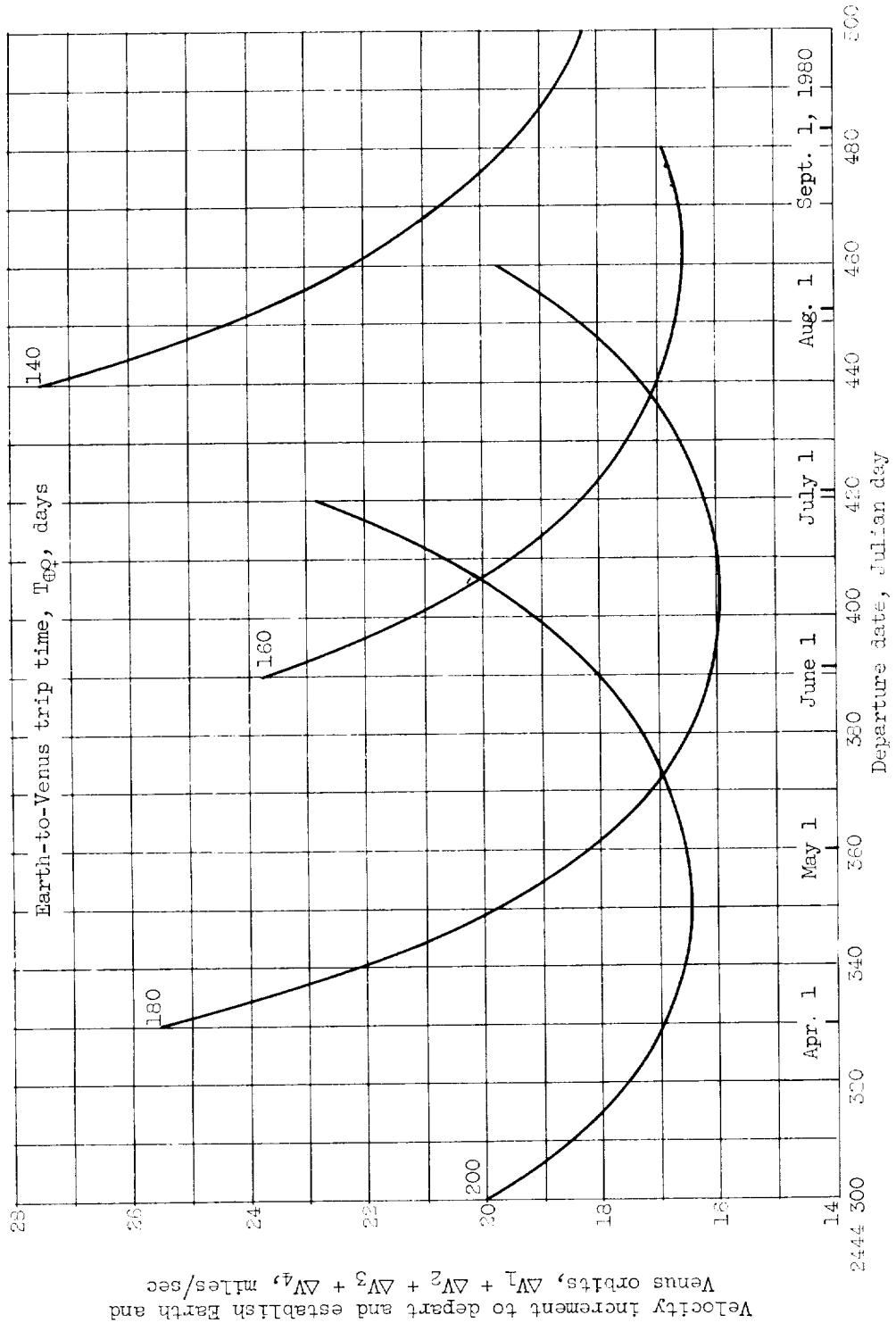
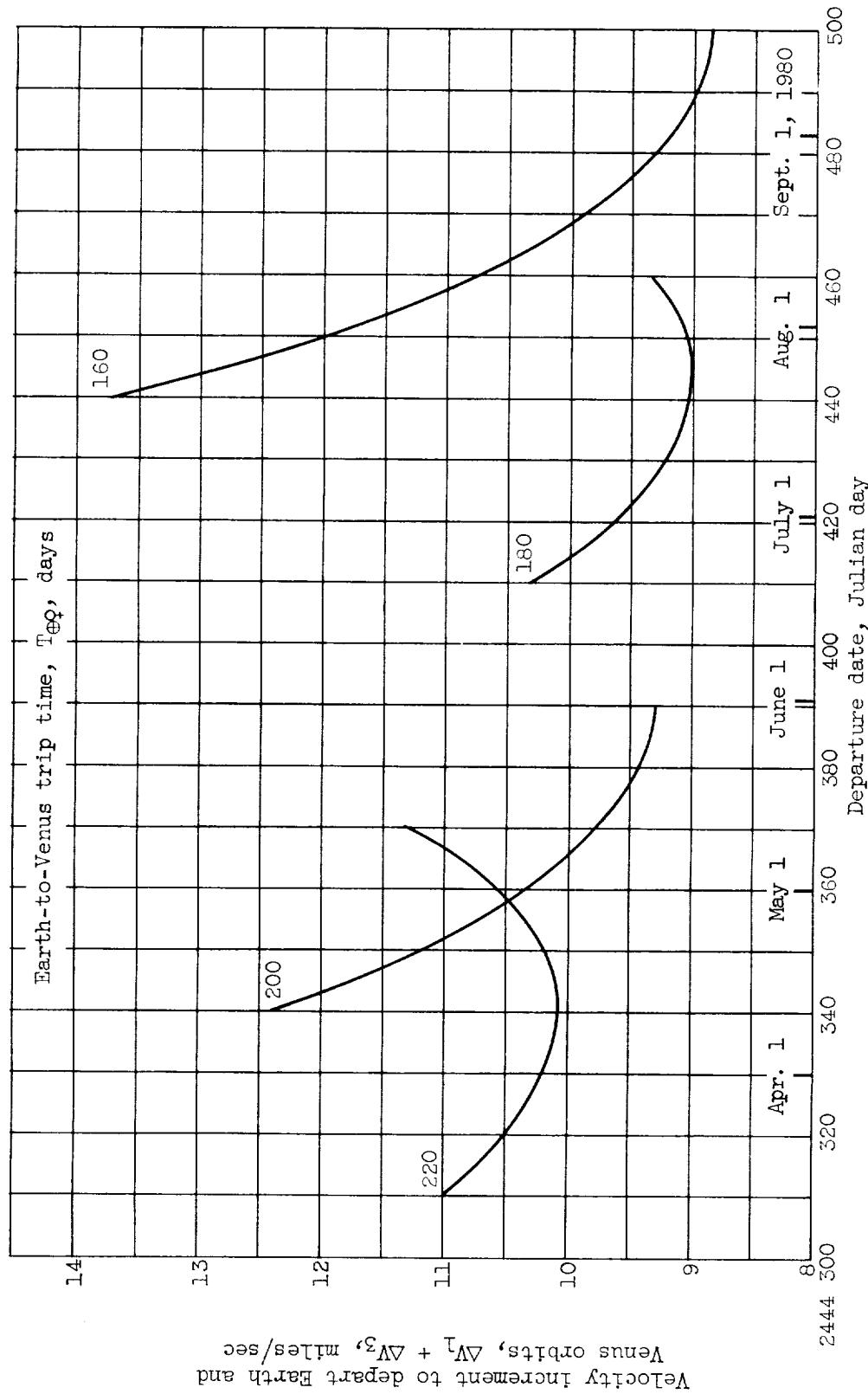


Figure 20. - Continued. Velocity increments for 550-day round trip to Venus. Wait time in Venus orbit, 200 days.



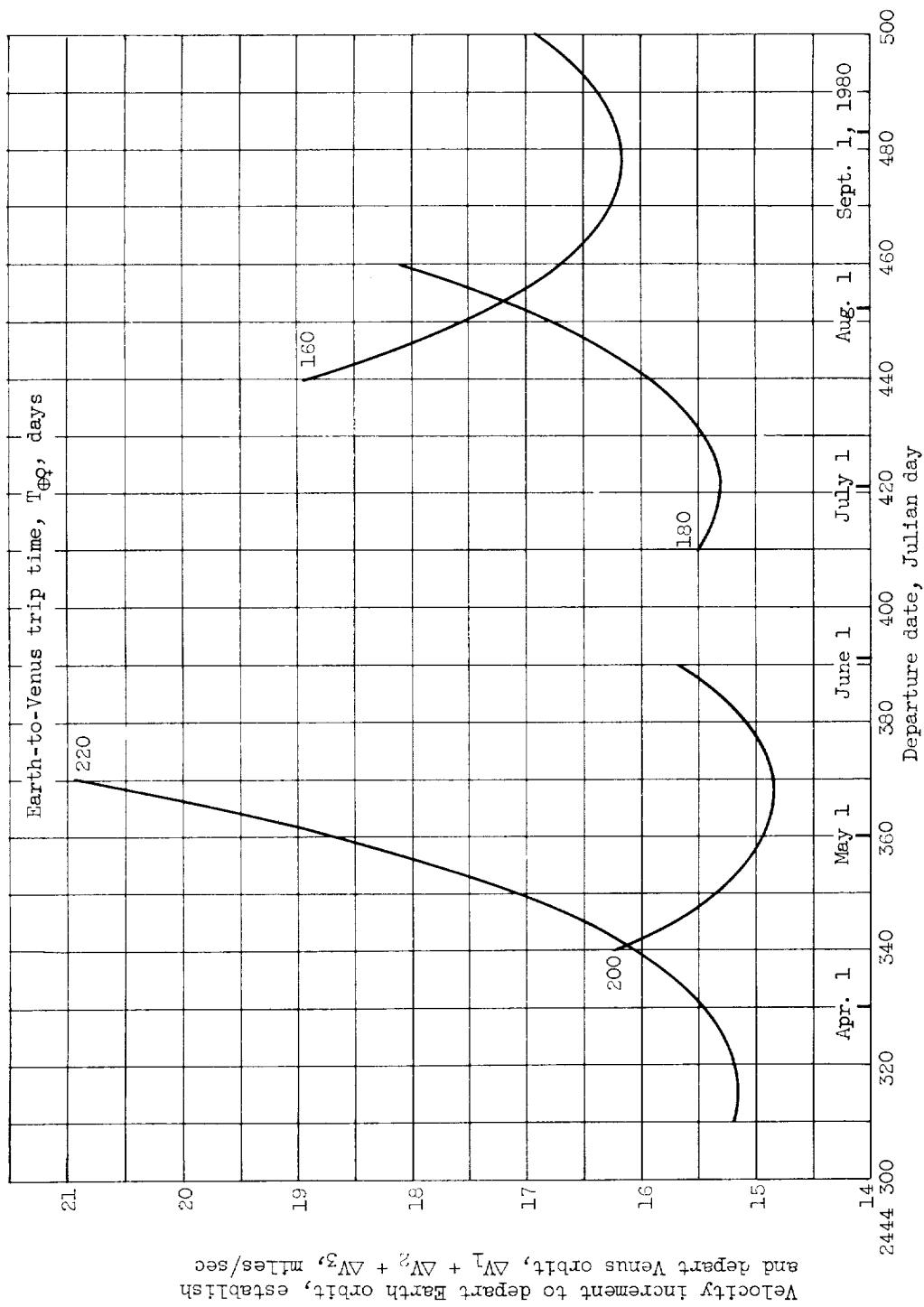
(c) All propulsive braking.

Figure 20. - Concluded. Velocity increments for 550-day round trip to Venus. Wait time in Venus orbit, 200 days.



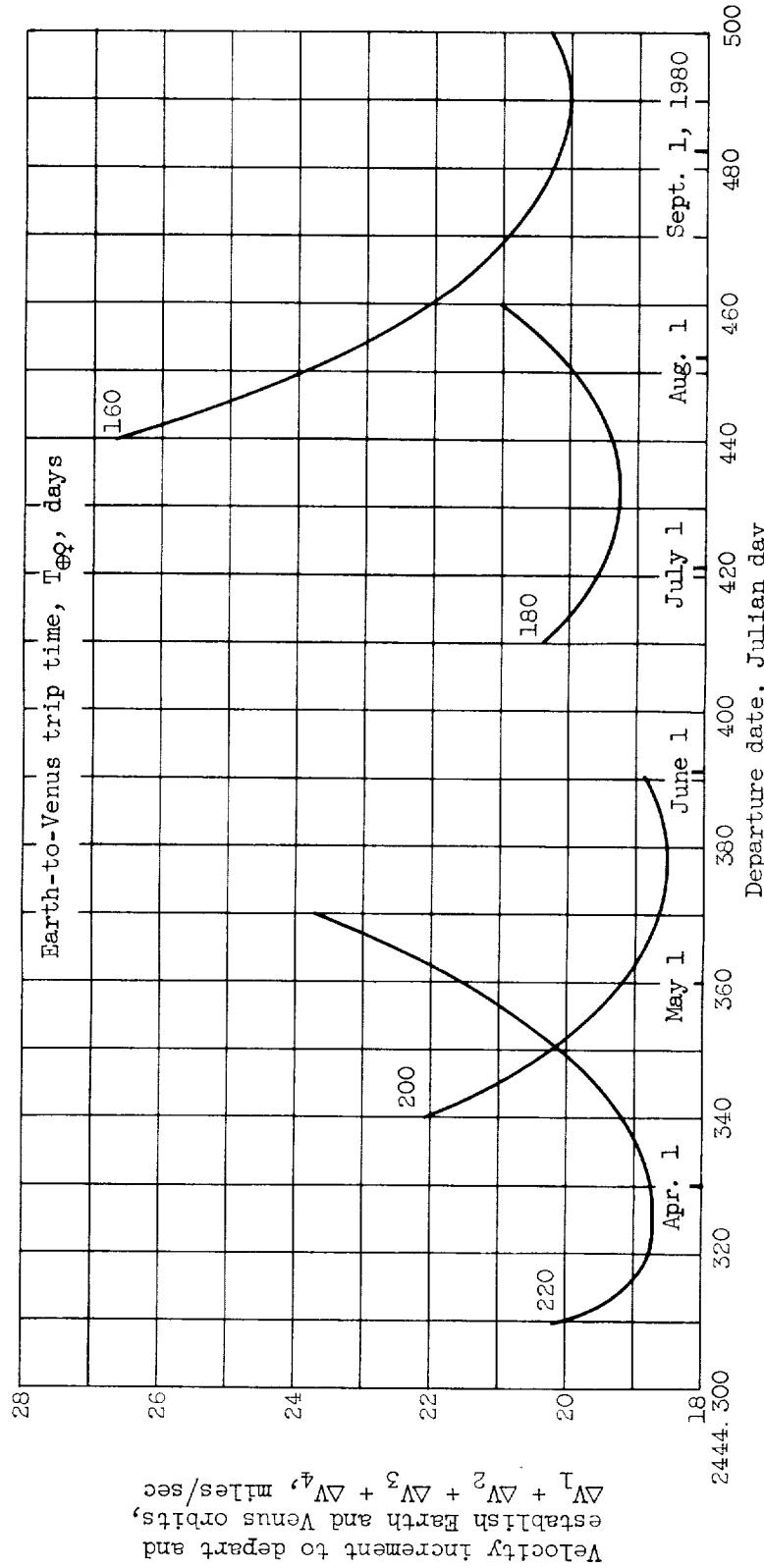
(a) Atmospheric braking at Venus and Earth.

Figure 21. - Velocity increments for 580-day round trip to Venus. Wait time in Venus orbit, 200 days.



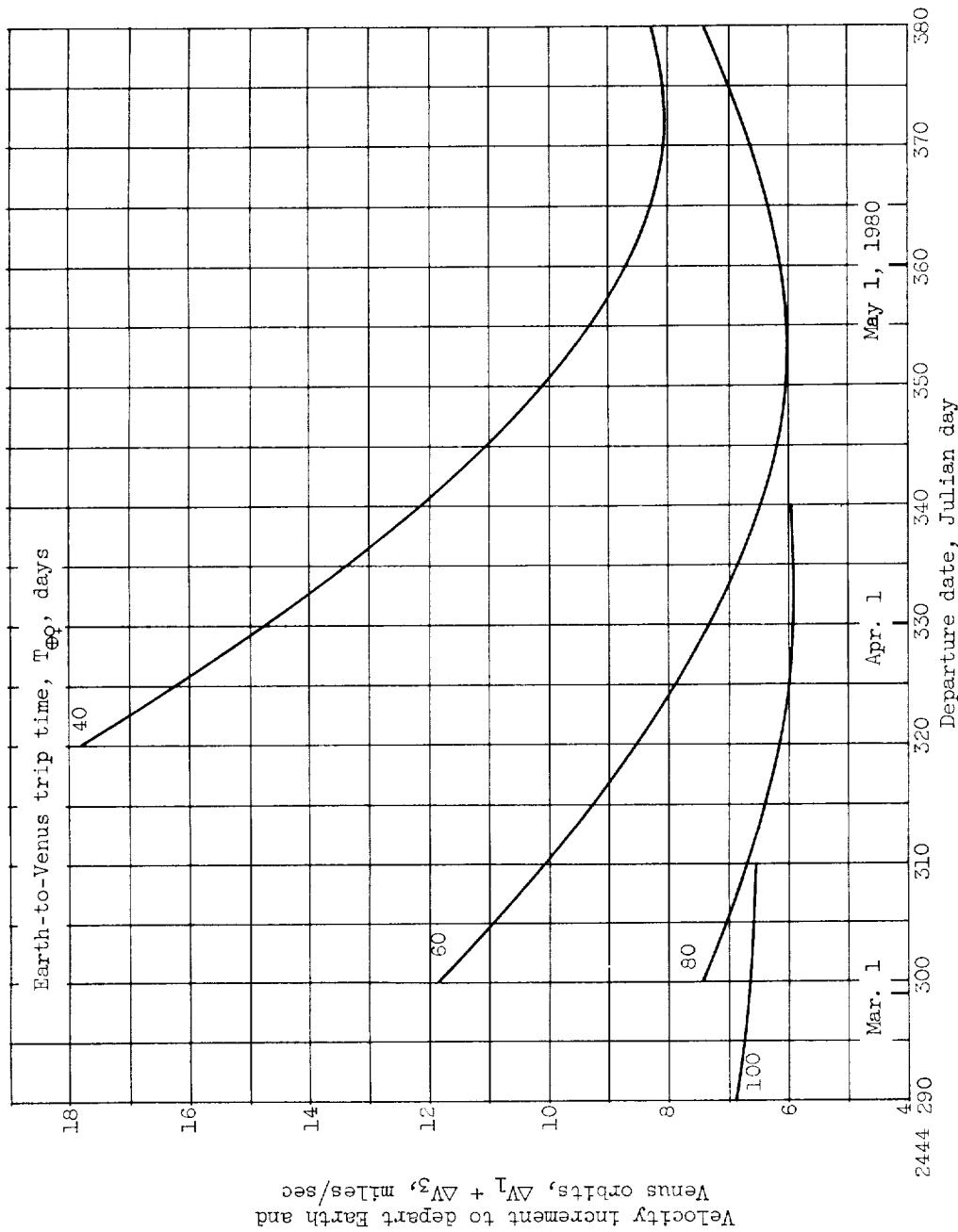
(b) Atmospheric braking at Earth.

Figure 21. - Continued. Velocity increments for 580-day round trip to Venus. Wait time in Venus orbit, 200 days.



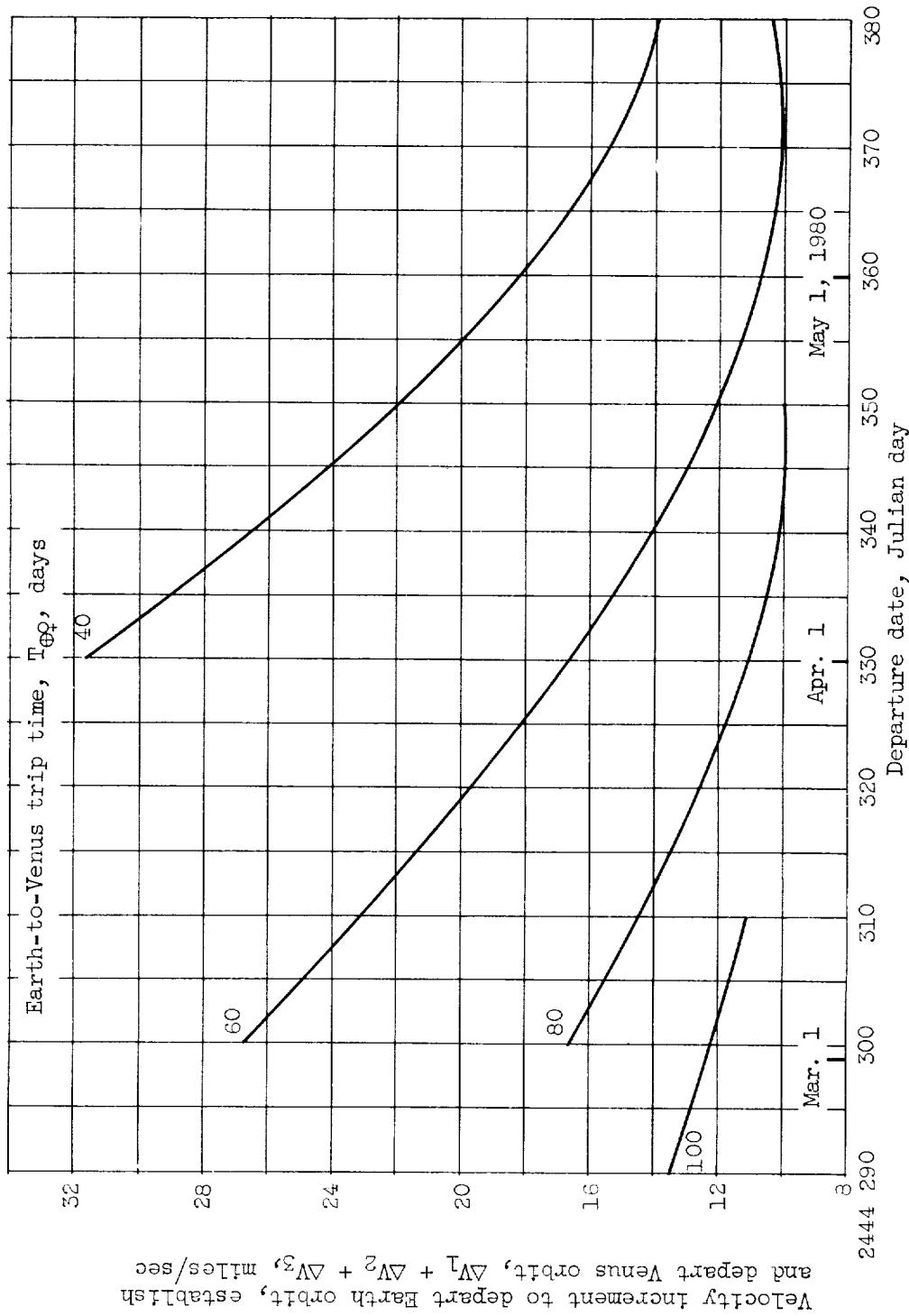
(c) All propulsive braking.

Figure 21. - Concluded. Velocity increments for 580-day round trip to Venus. Wait time in Venus orbit, 200 days.



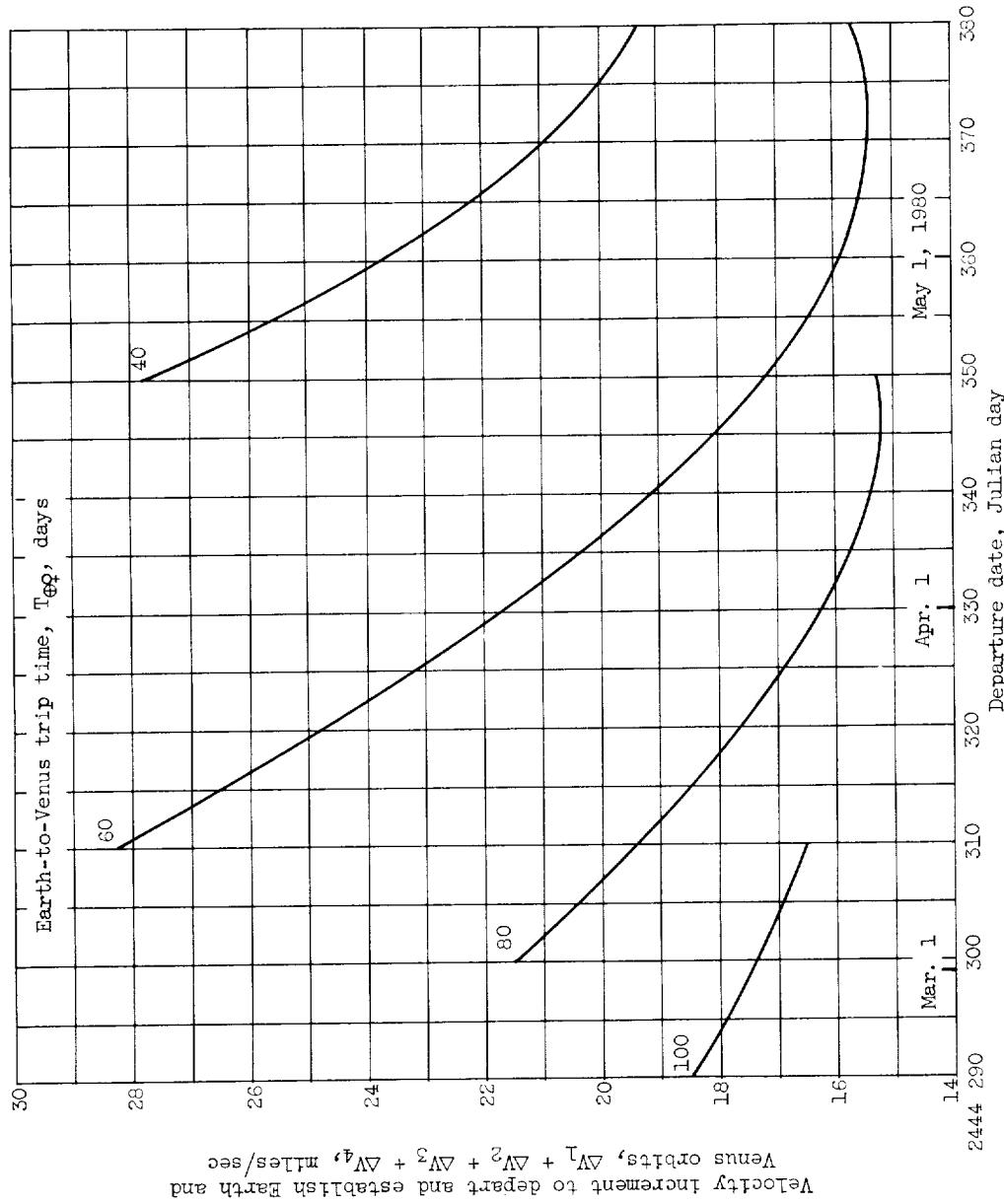
(a) Atmospheric braking at Venus and Earth.

Figure 22. - Velocity increments for 660-day round trip to Venus. Wait time in Venus orbit, 200 days.



(b) Atmospheric braking at Earth.

Figure 22. - Continued. Velocity increments for 660-day round trip to Venus. Wait time in Venus orbit, 260 days.



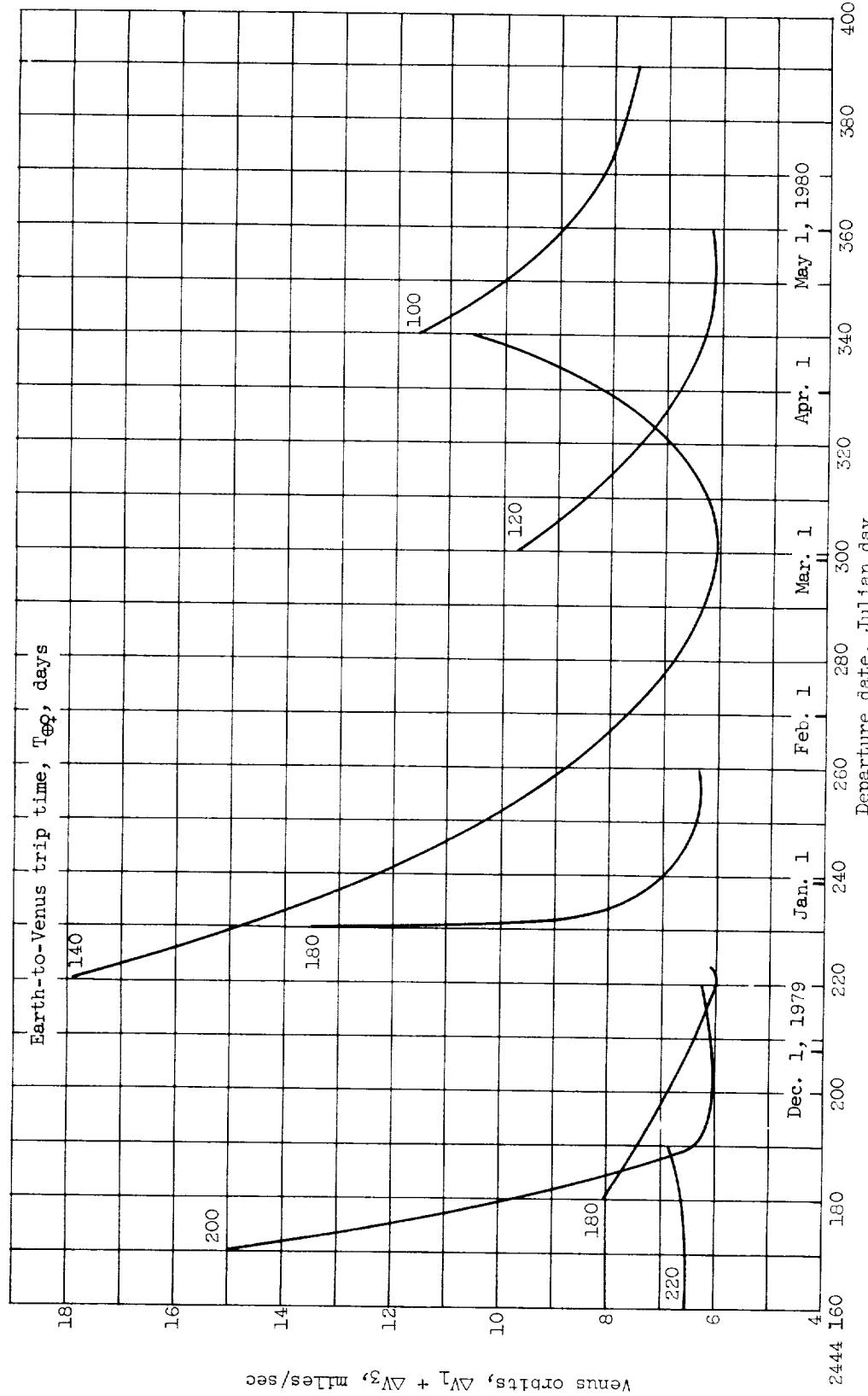
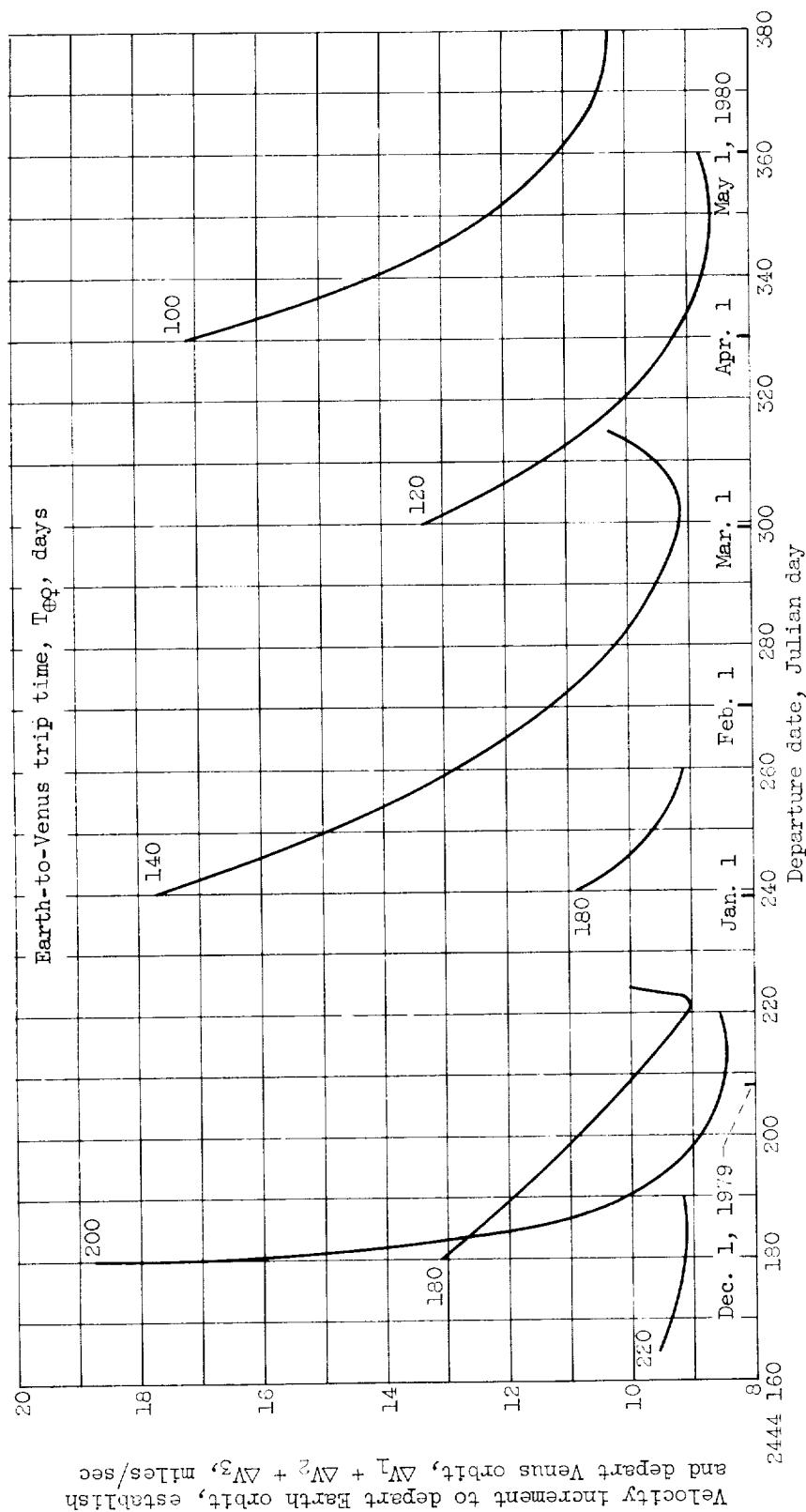


Figure 23. - Velocity increments for 780-day round trip to Venus. Wait time in Venus orbit, 200 days.
 (a) Atmospheric braking at Venus and Earth.



(b) Atmospheric braking at Earth.

Figure 23. - Continued. Velocity increments for 760-day round trip to Venus. Wait time in Venus orbit, 200 days.

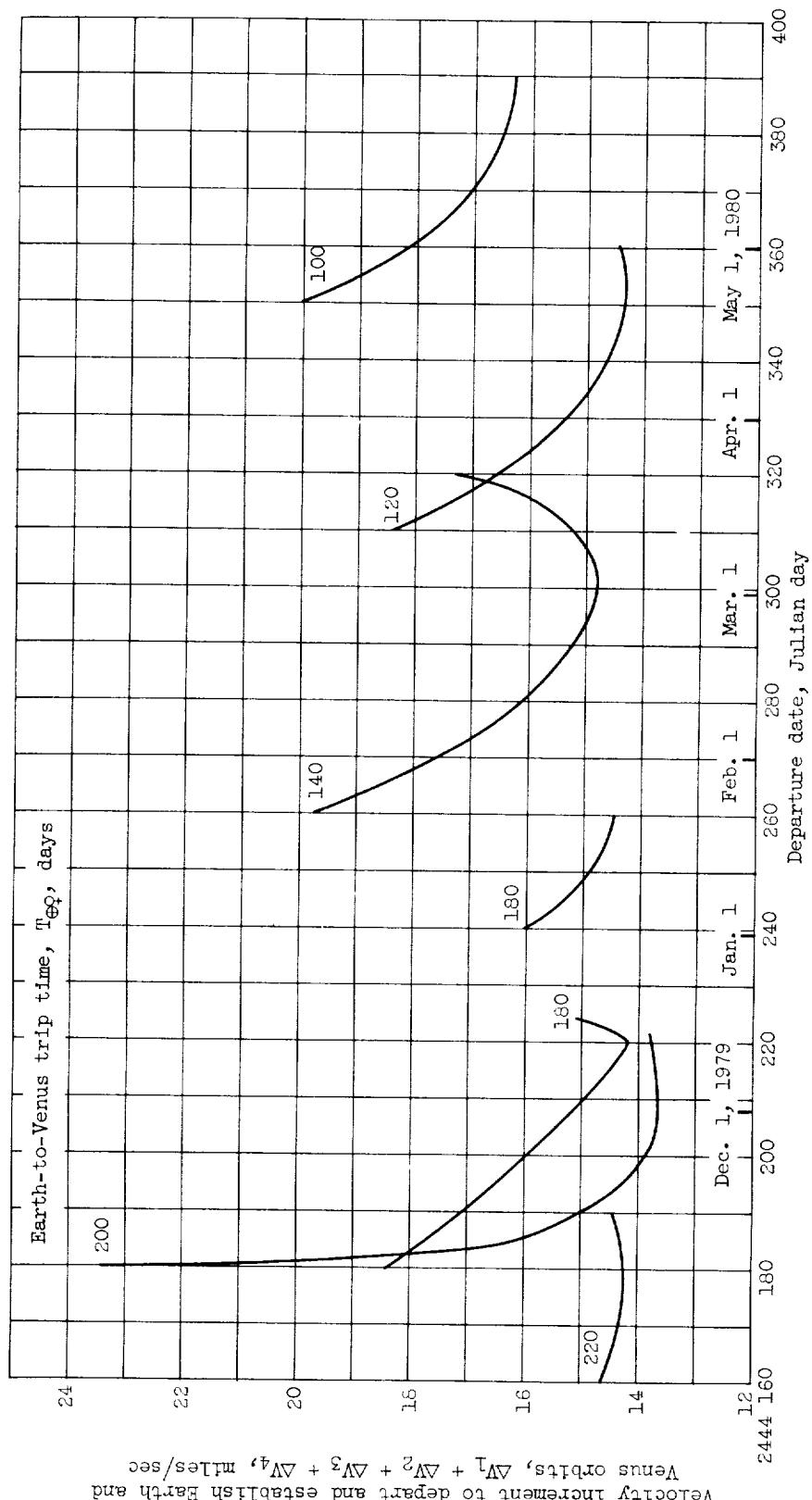
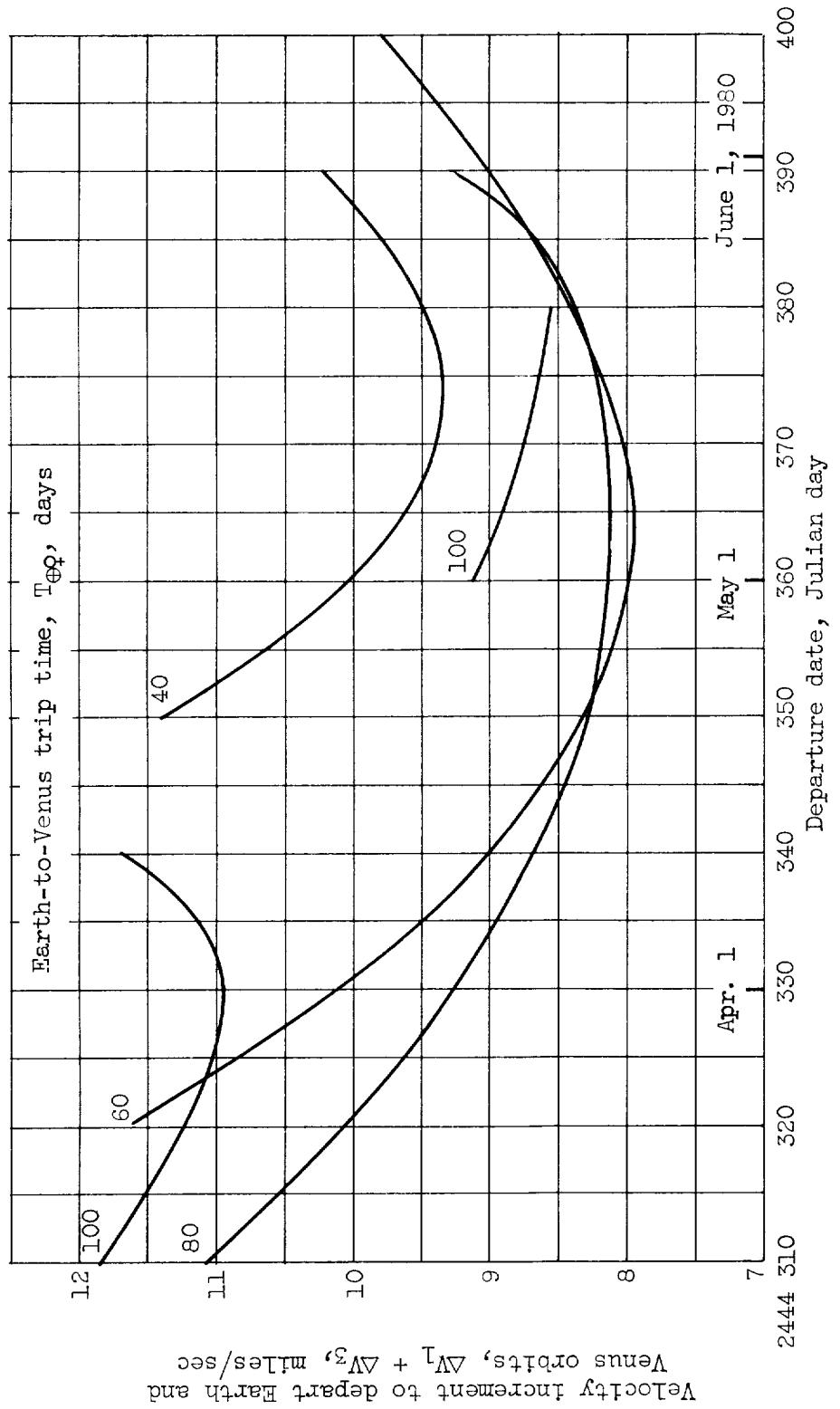


Figure 23. - Concluded. Velocity increments for 780-day round trip to Venus. Wait time in Venus orbit, 200 days.



(a) Atmospheric braking at Venus and Earth.

Figure 24. - Velocity increments for 580-day round trip to Venus. Wait time in Venus orbit, 390 days.

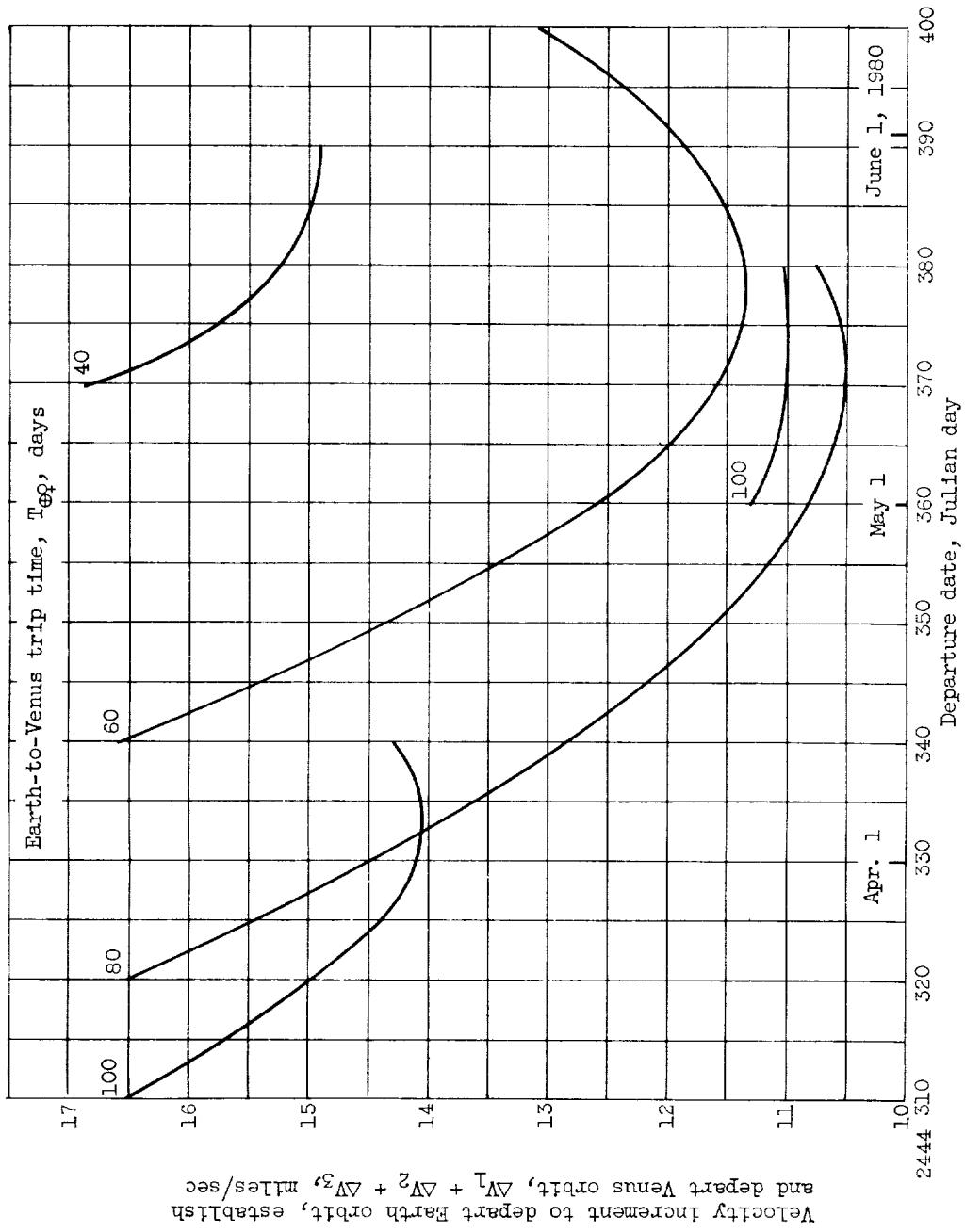
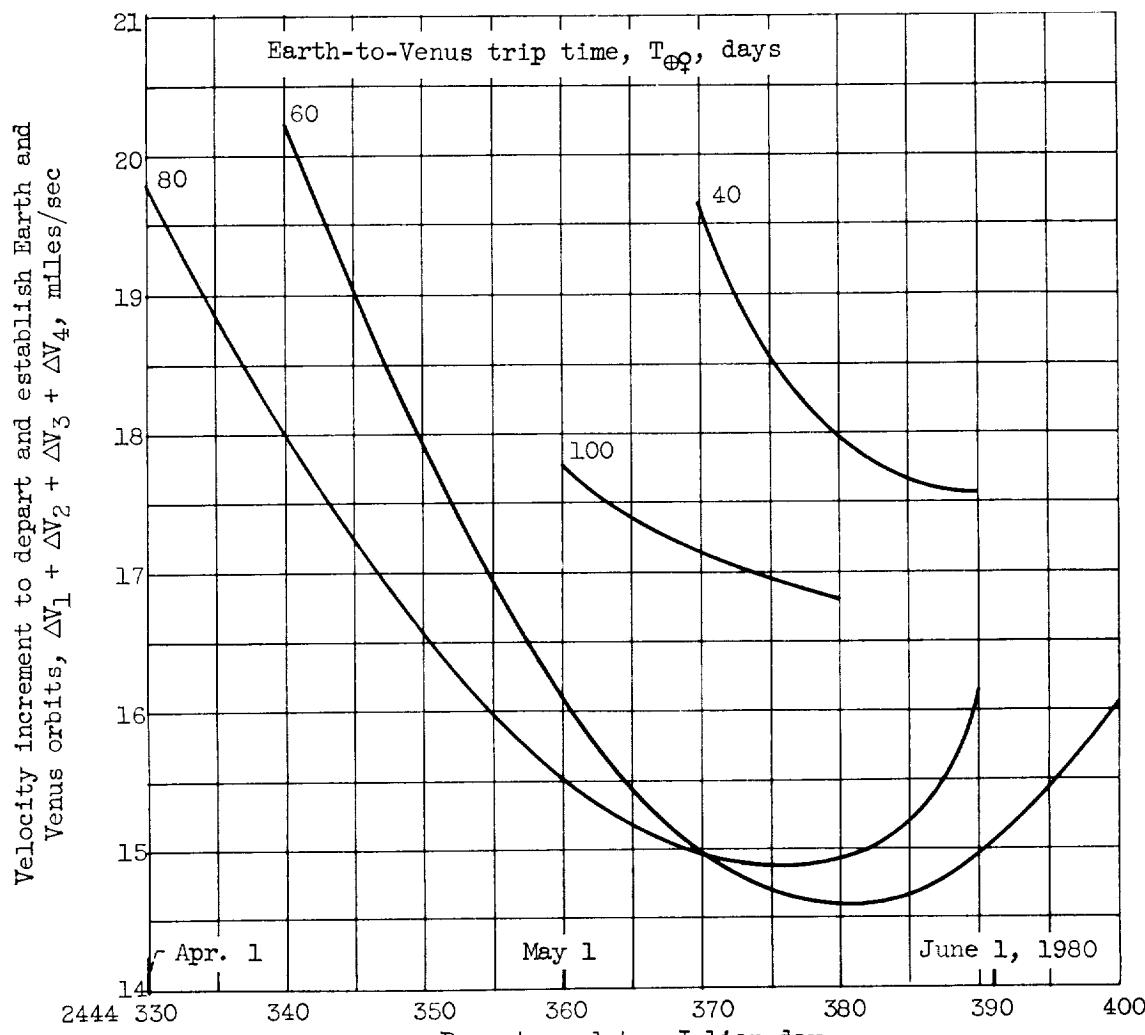
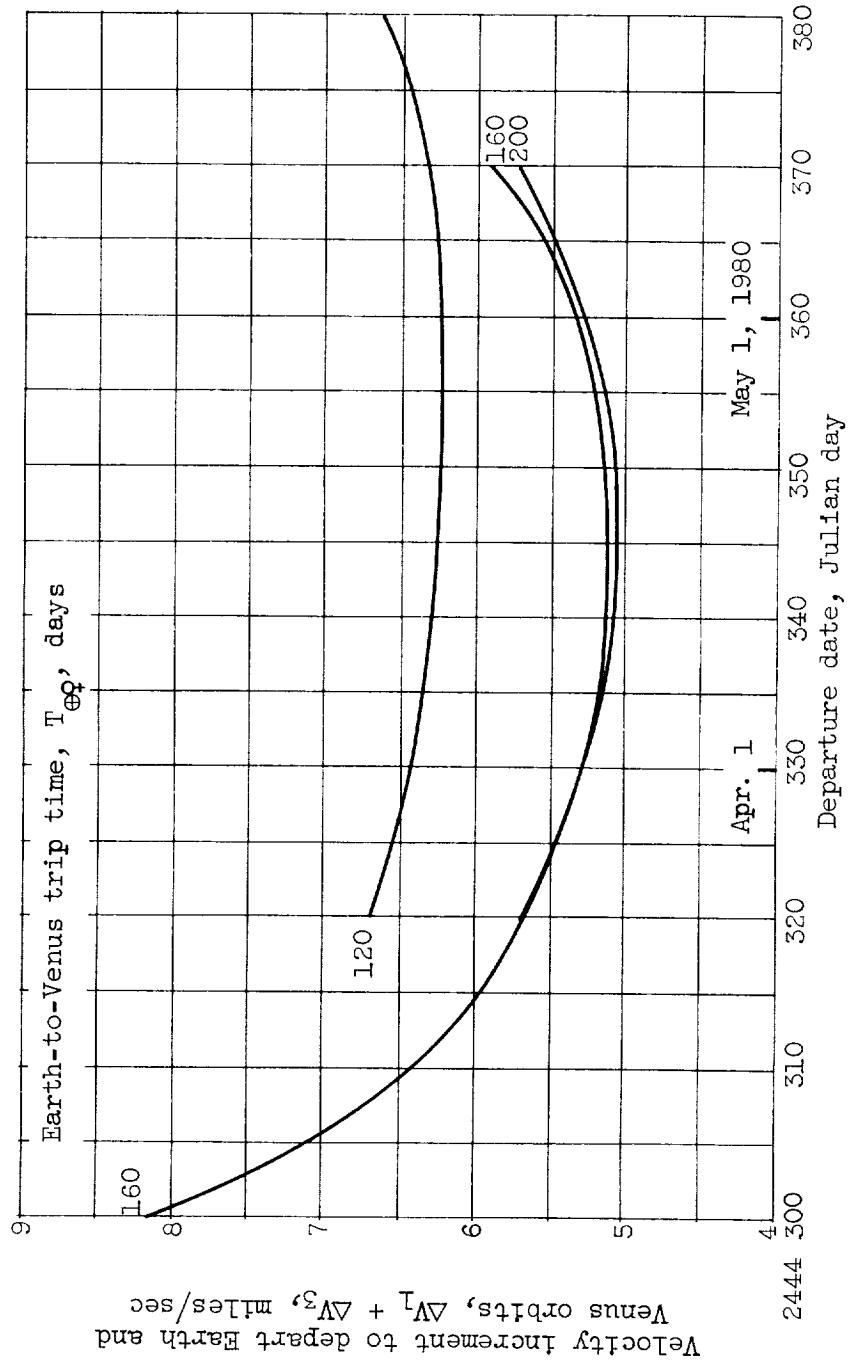


Figure 24. - Continued. Velocity increments for 580-day round trip to Venus. Wait time in Venus orbit, 390 days.



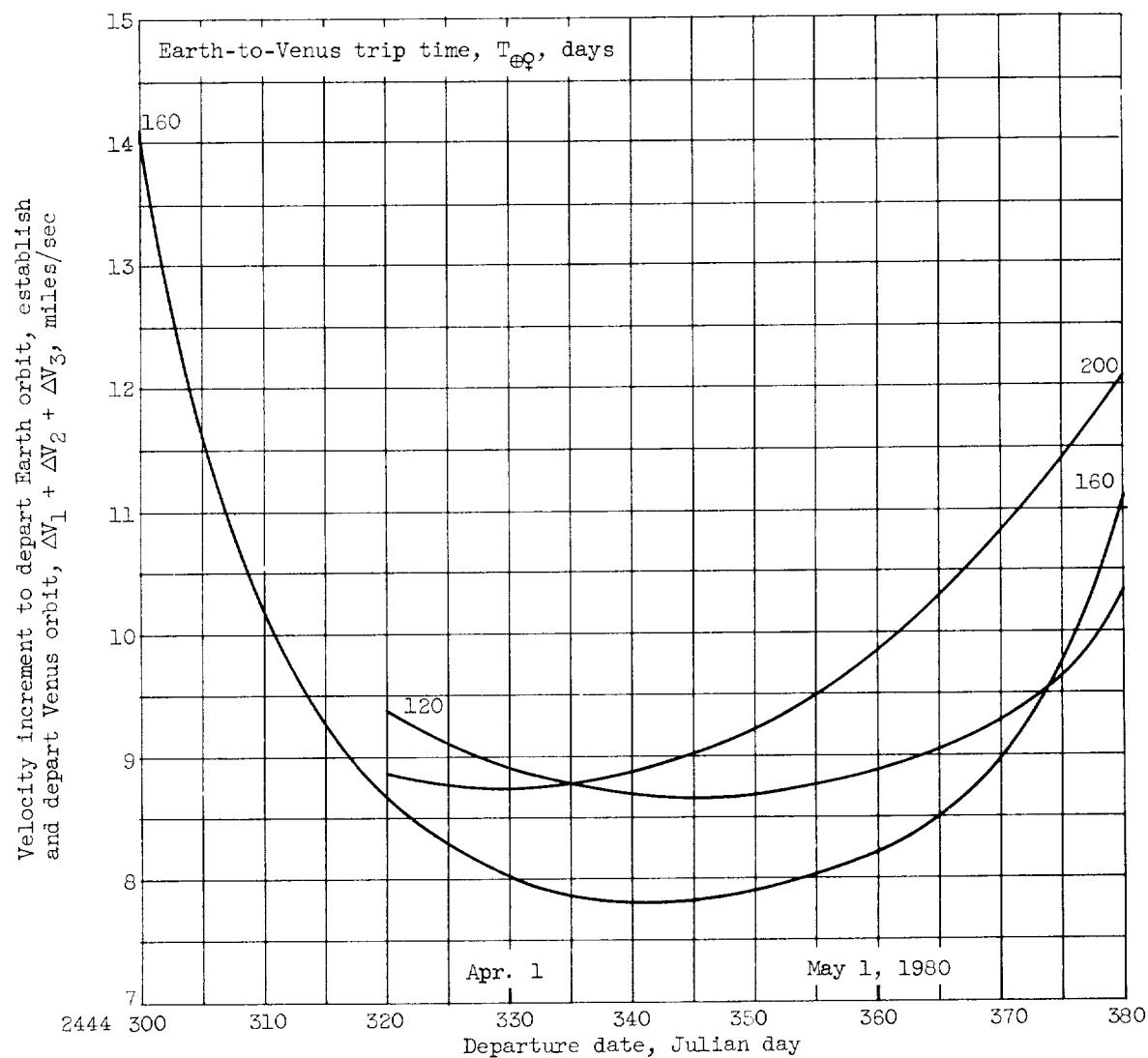
(c) All propulsive braking.

Figure 24. - Concluded. Velocity increments for 580-day round trip to Venus. Wait time in Venus orbit, 390 days.



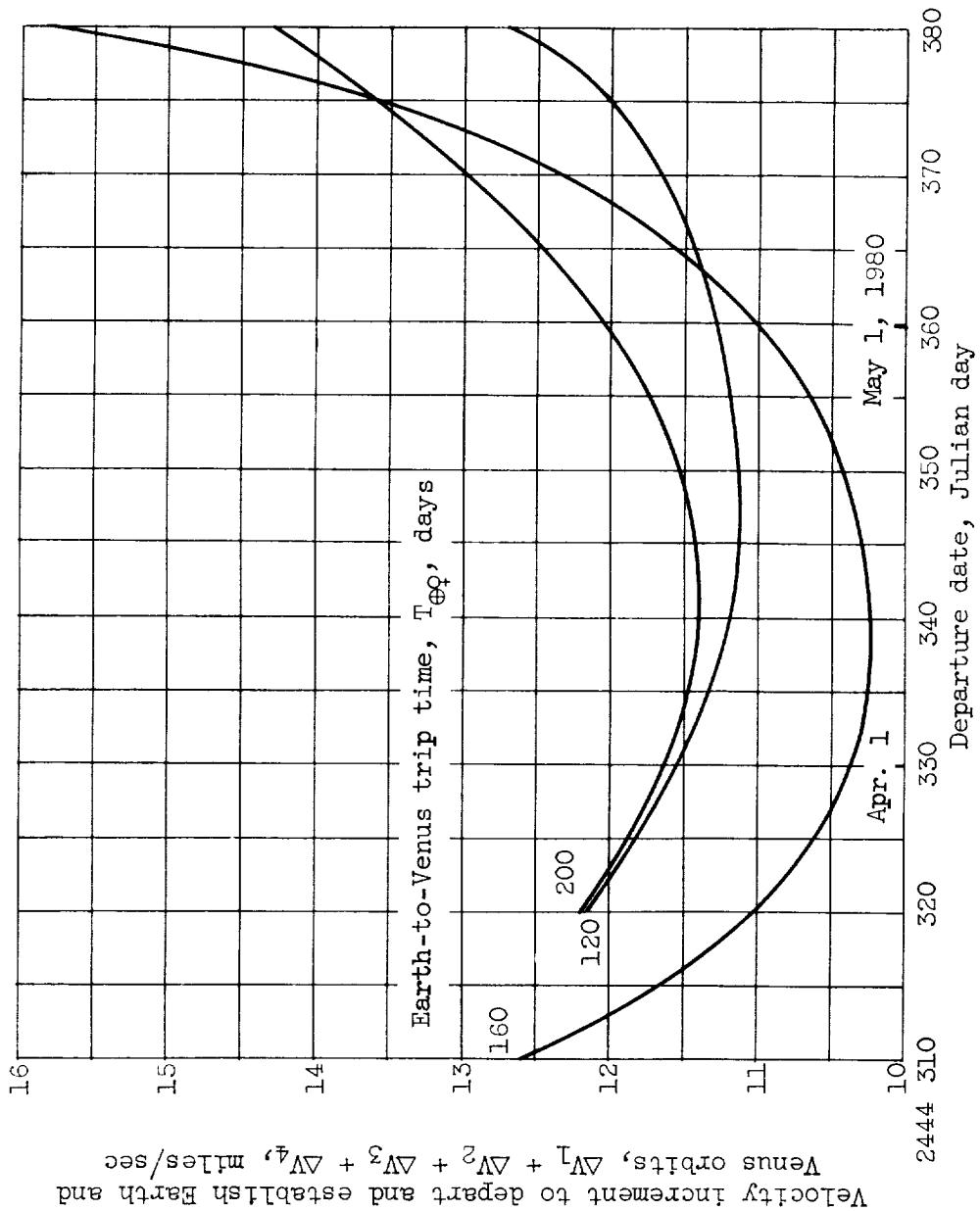
(a) Atmospheric braking at Venus and Earth.

Figure 25. - Velocity increments for 700-day round trip to Venus. Wait time in Venus orbit, 390 days.



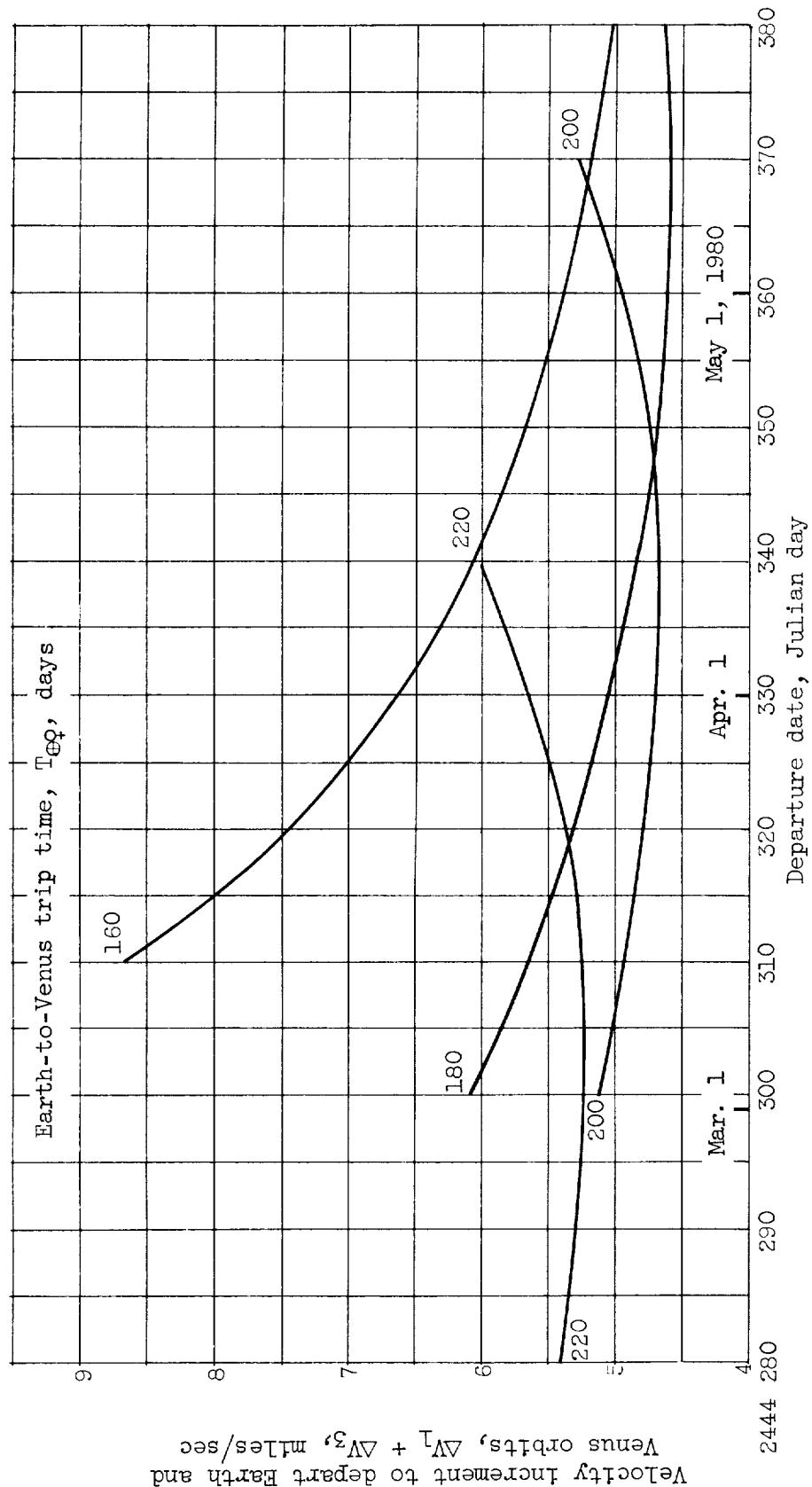
(b) Atmospheric braking at Earth.

Figure 25. - Continued. Velocity increments for 700-day round trip to Venus.
Wait time in Venus orbit, 390 days.



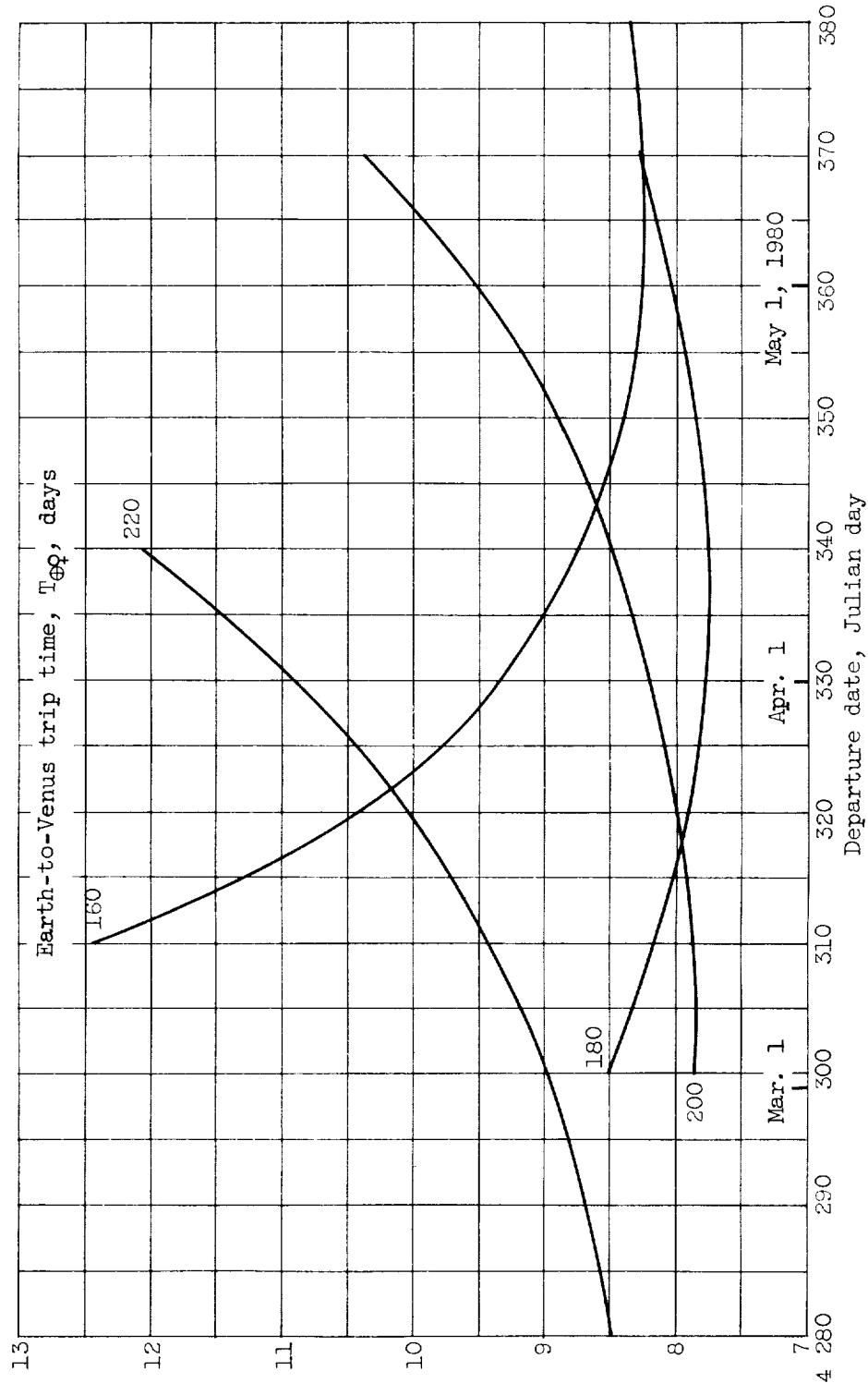
(c) All propulsive braking.

Figure 25. - Concluded. Velocity increments for 700-day round trip to Venus. Wait time in Venus orbit, 390 days.



(a) Atmospheric braking at Venus and Earth.

Figure 26. - Velocity increments for 780-day round trip to Venus. Wait time in Venus orbit, 390 days.



Velocity increment to depart Earth orbit, $\Delta V^1 + \Delta V^2 + \Delta V^3$, miles/sec
and depart Venus orbit, ΔV^1 , establish orbit, to increment velocity to depart Earth.

(b) Atmospheric braking at Earth.

Figure 26. - Continued. Velocity increments for 780-day round trip to Venus. Wait time in Venus orbit, 390 days.

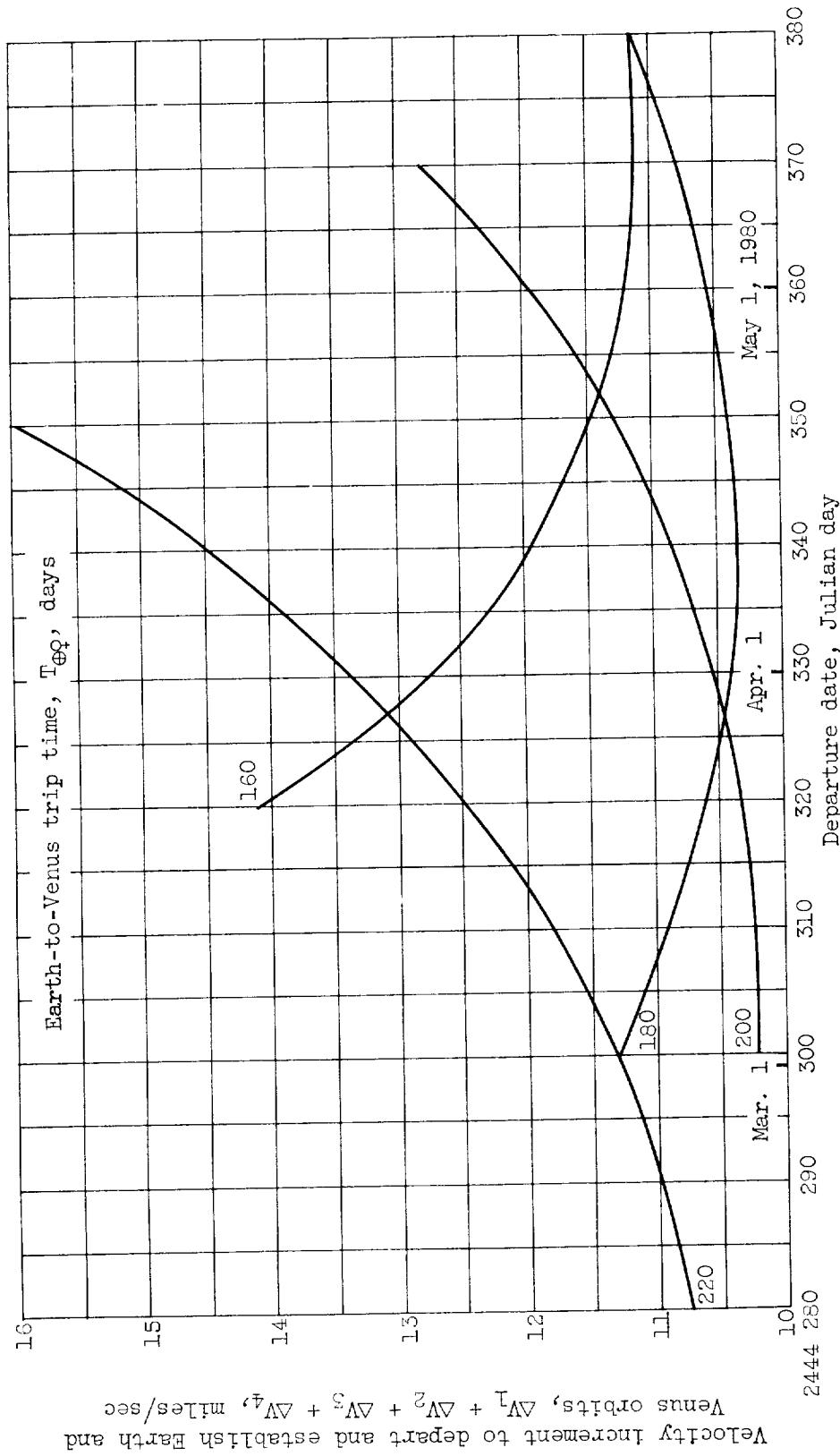


Figure 26. - Concluded. Velocity increments for 780-day round trip to Venus. Wait time in Venus orbit, 390 days.

(c) All propulsive braking.

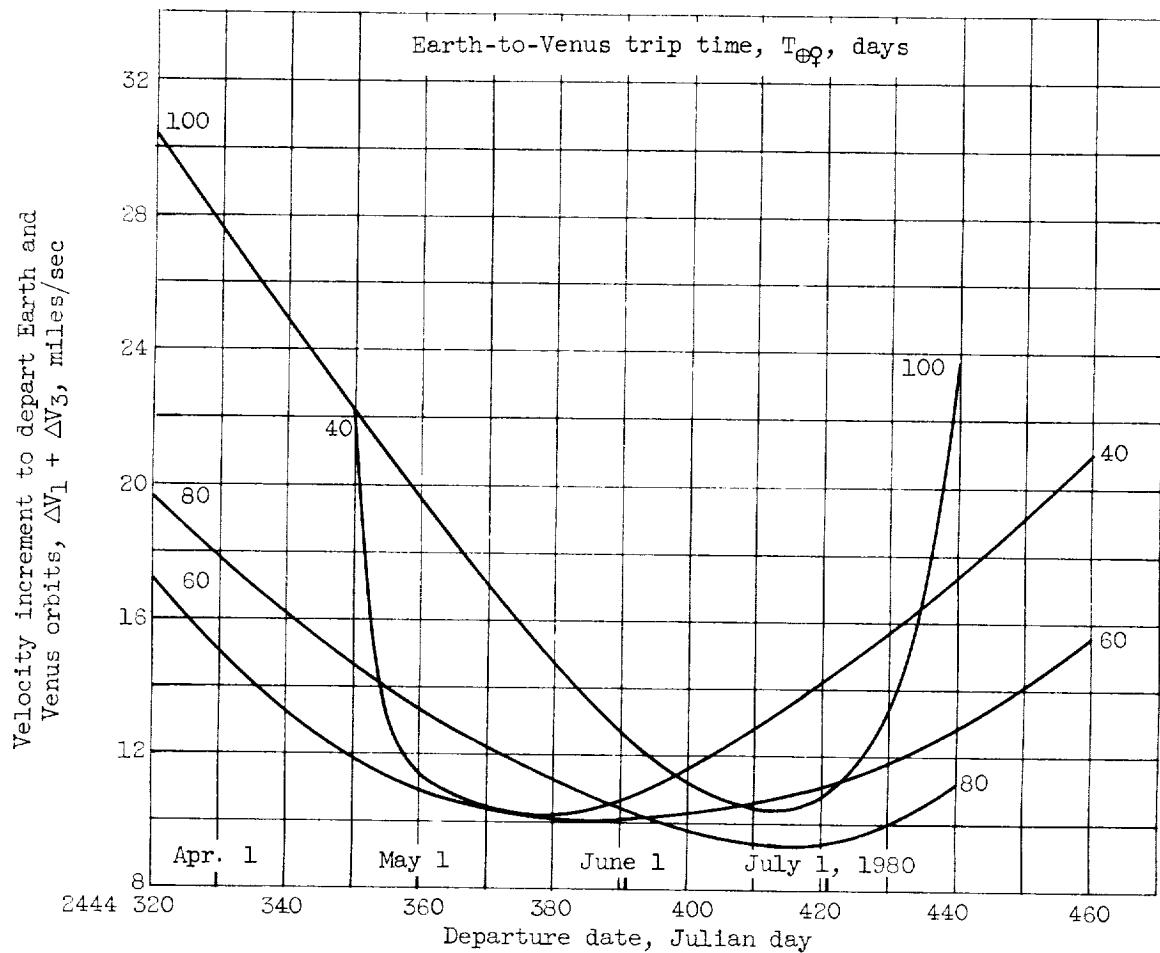
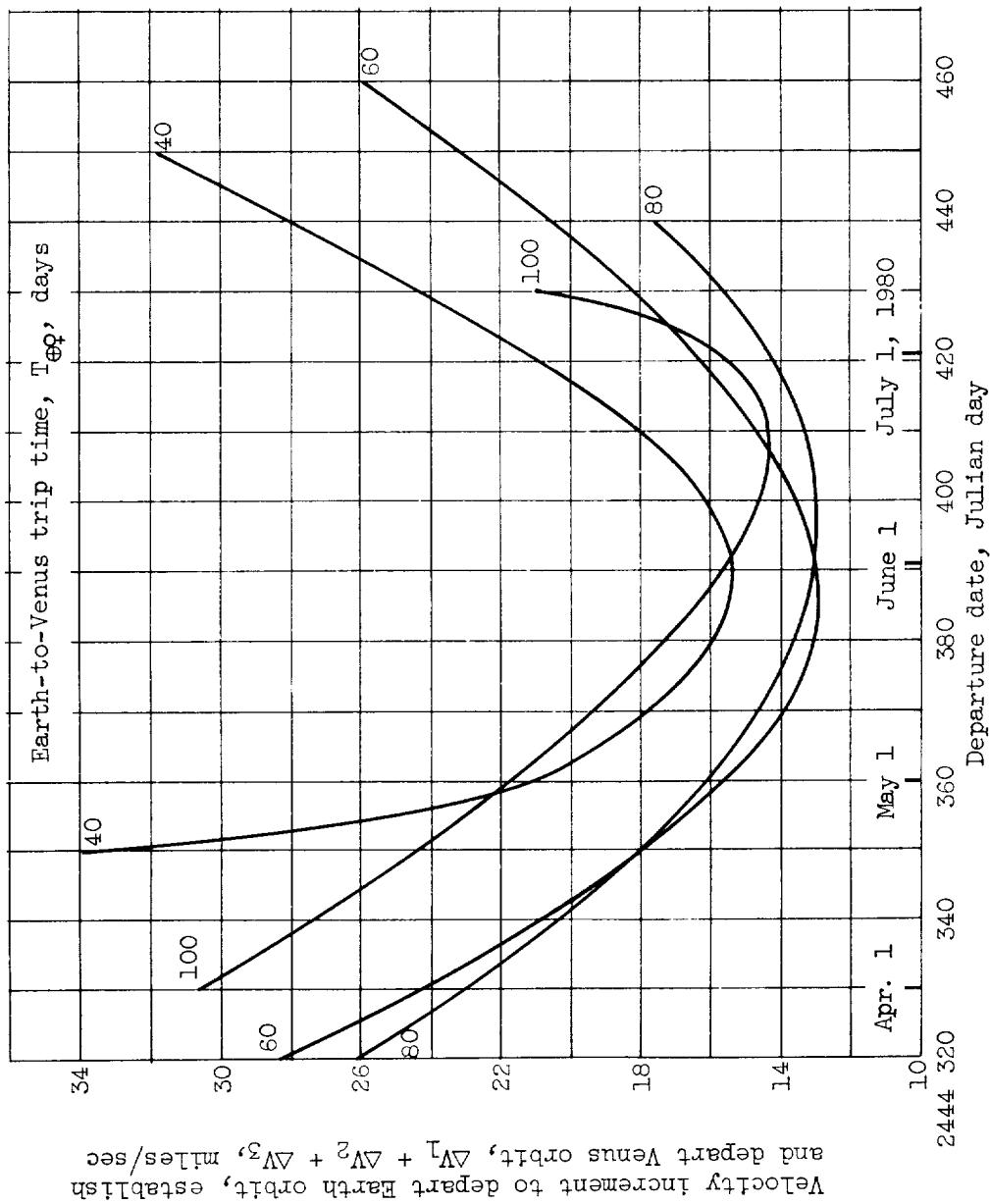
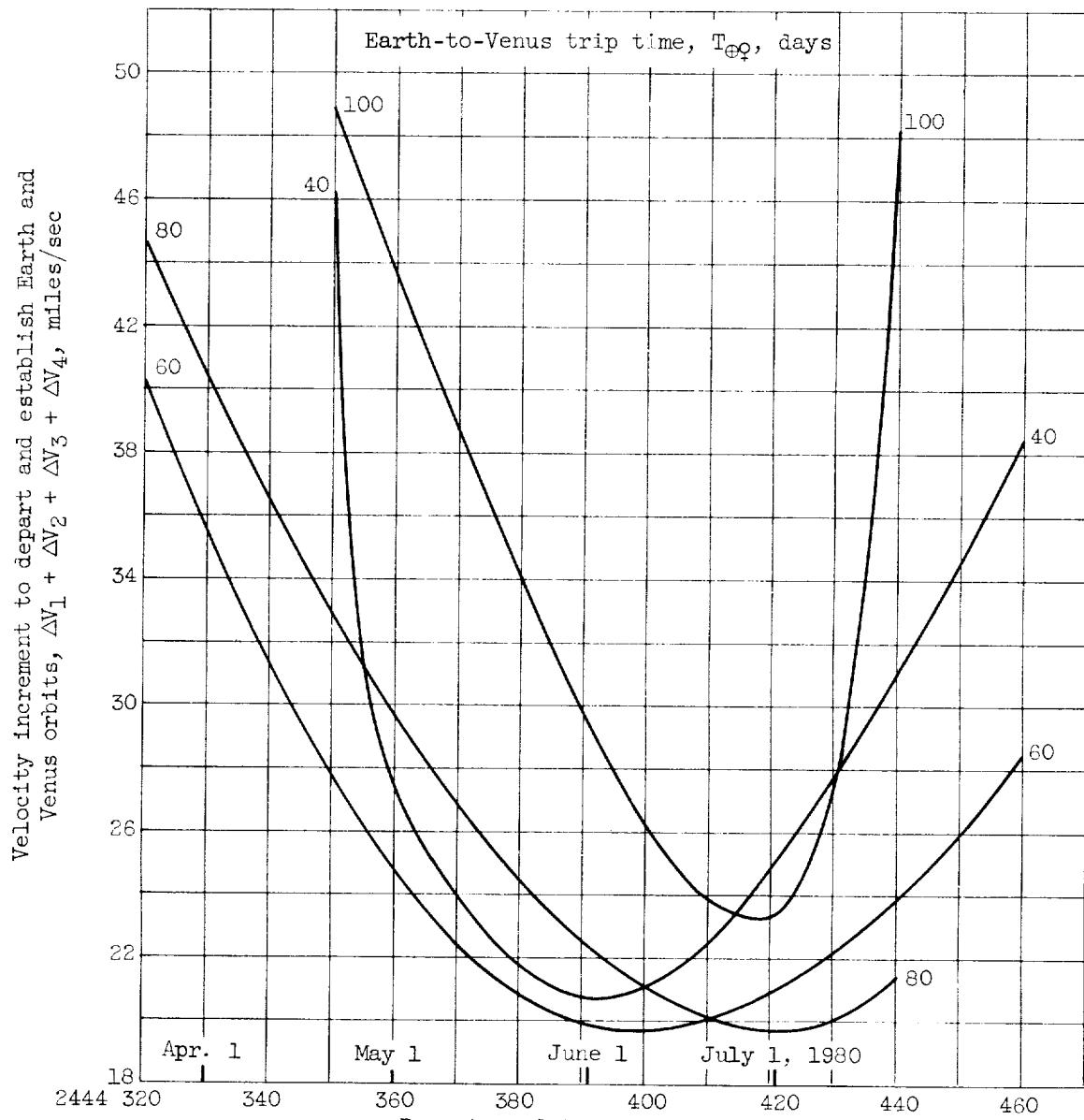


Figure 27. - Velocity increments for 580-day round trip to Venus. Wait time in Venus orbit, 450 days.



(b) Atmospheric braking at Earth.

Figure 27. - Continued. Velocity increments for 580-day round trip to Venus.
Wait time in Venus orbit, 450 days.



(c) All propulsive braking.

Figure 27. - Concluded. Velocity increments for 580-day round trip to Venus. Wait time in Venus orbit, 450 days.

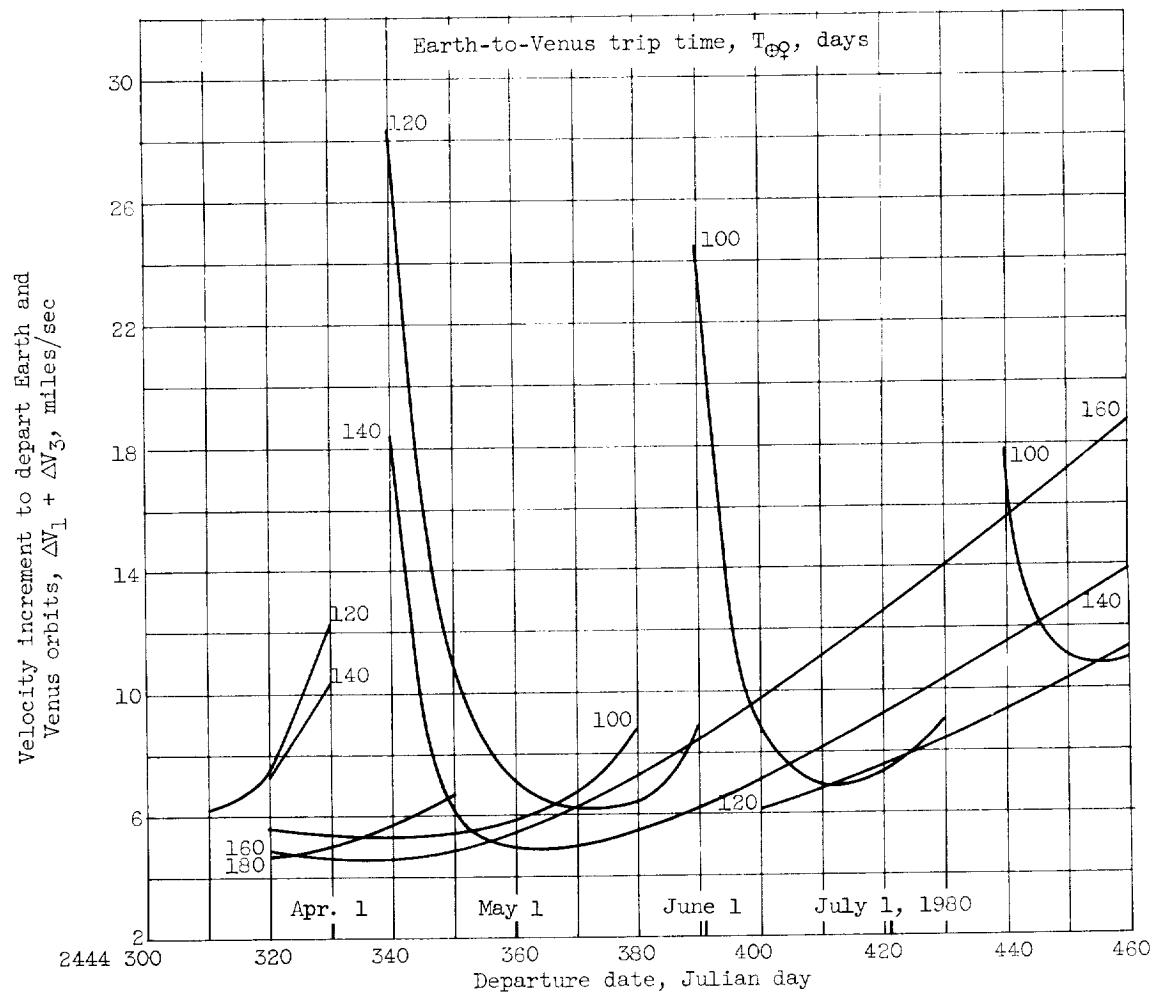


Figure 28. - Velocity increments for 700-day round trip to Venus. Wait time in Venus orbit, 450 days.

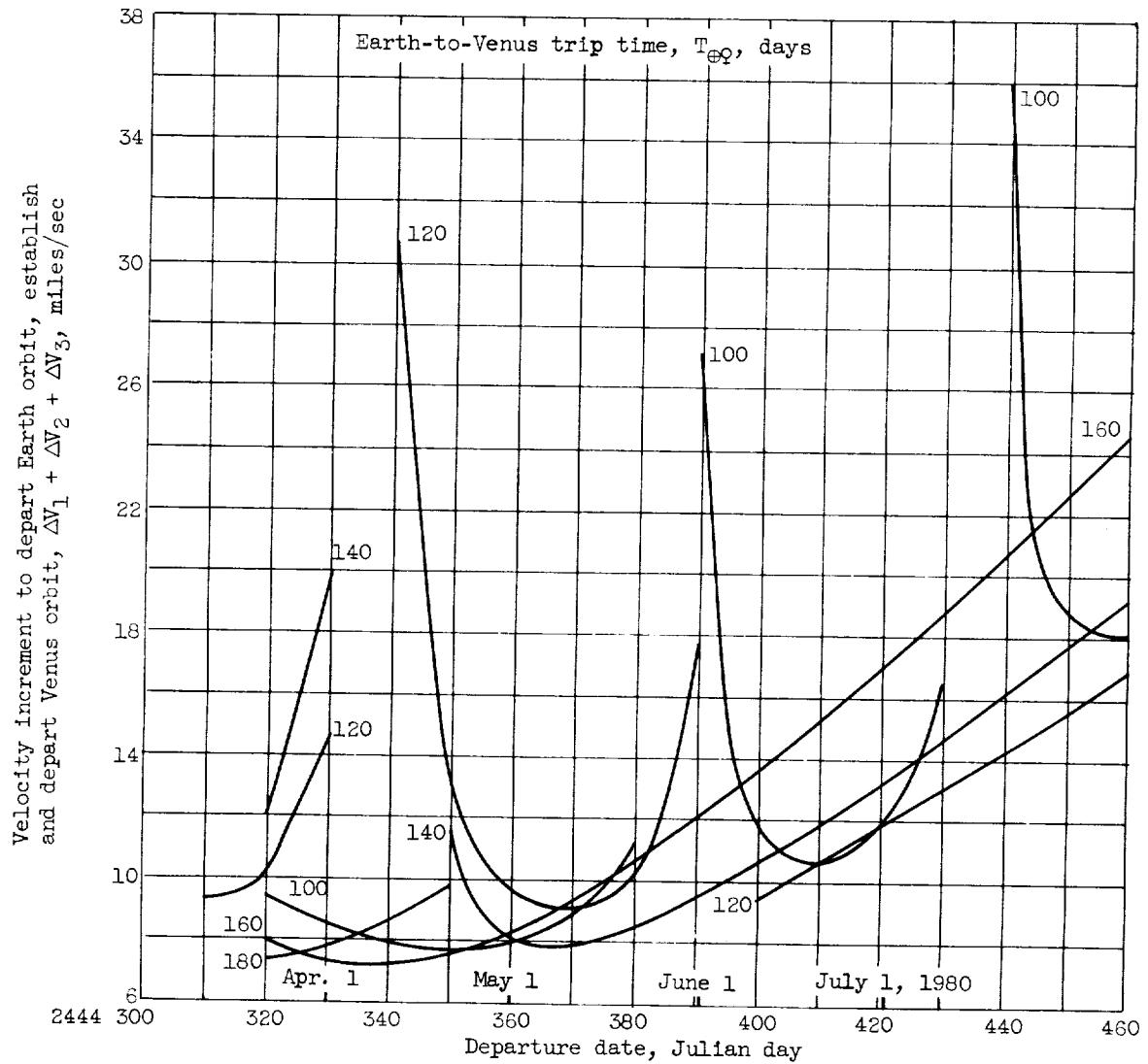
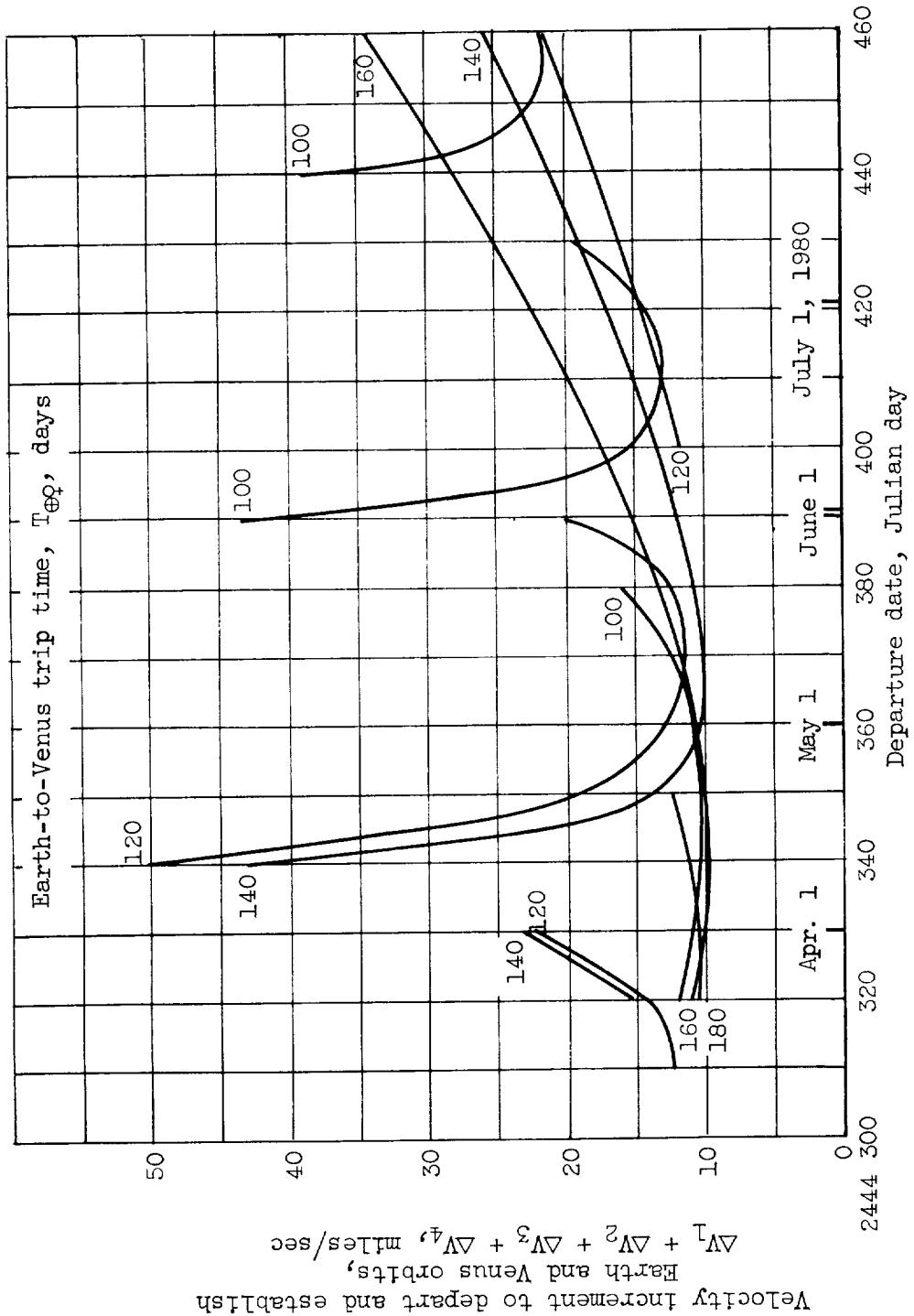
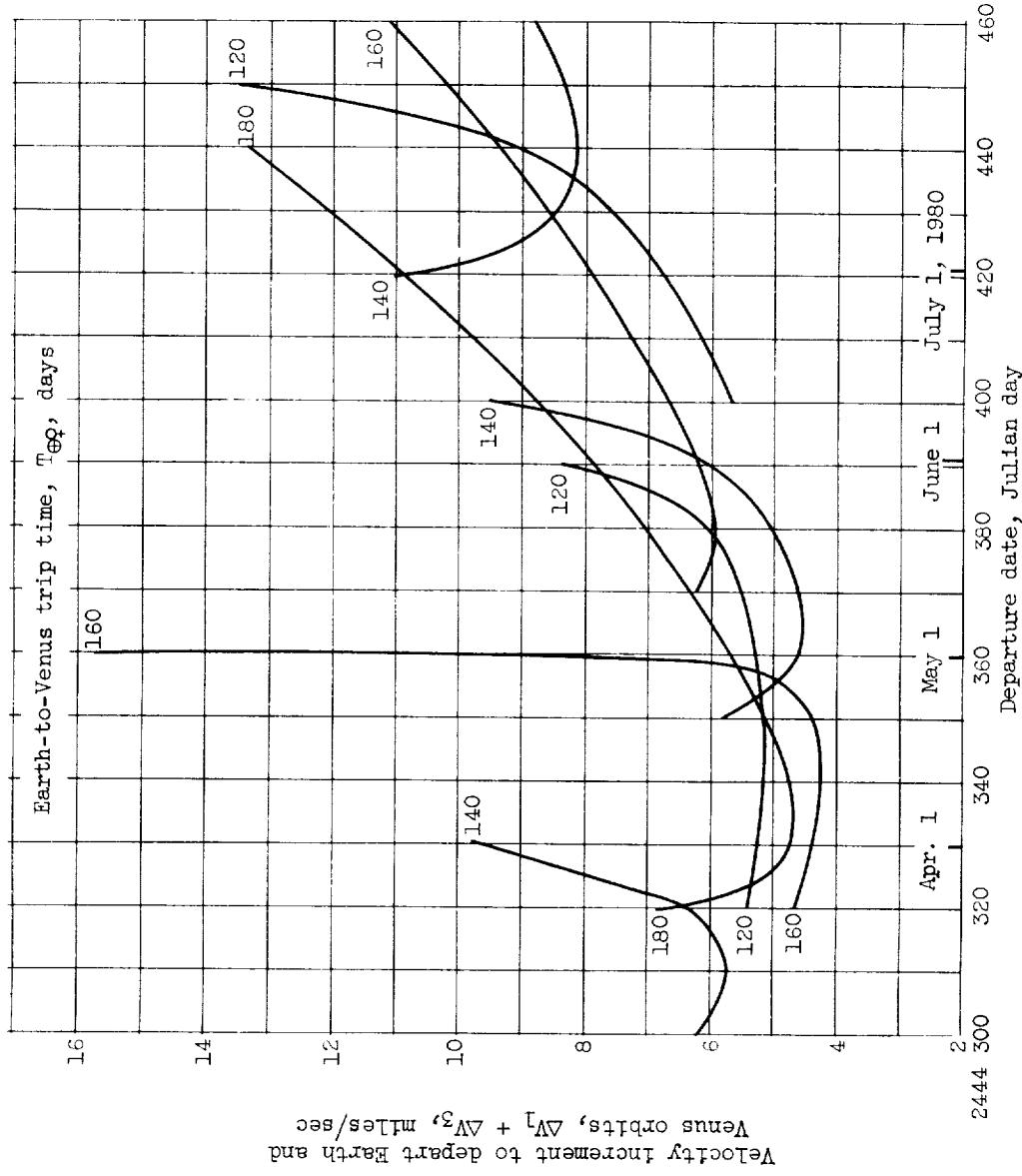


Figure 28. - Continued. Velocity increments for 700-day round trip to Venus.
Wait time in Venus orbit, 450 days.



(c) All propulsive braking.

Figure 28. - Concluded. Velocity increments for 700-day round trip to Venus.
 Wait time in Venus orbit, 450 days.



(a) Atmospheric braking at Venus and Earth.

Figure 29. - Velocity increments for 780-day round trip to Venus. Wait time in Venus orbit, 450 days.

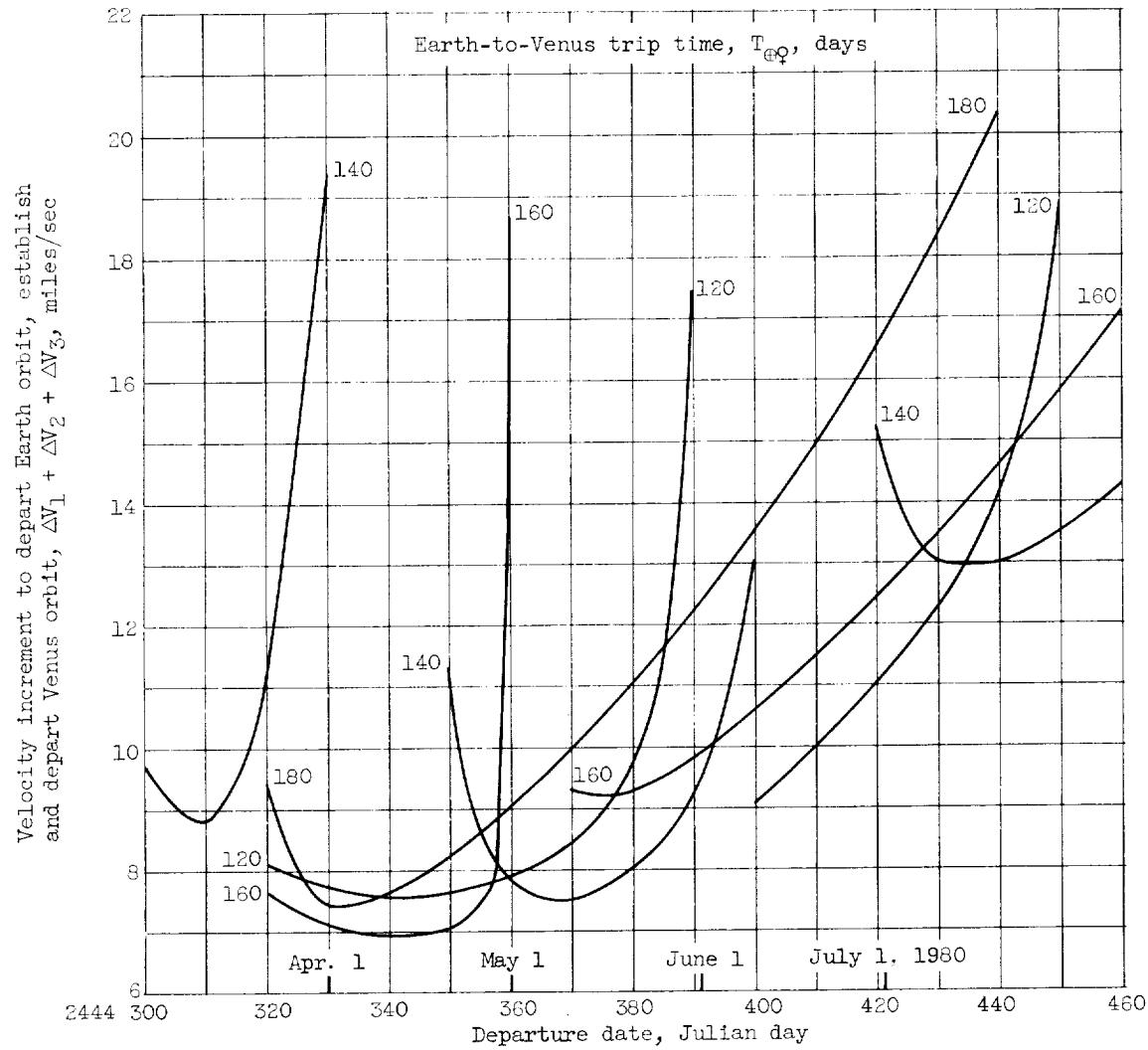


Figure 29. - Continued. Velocity increments for 780-day round trip to Venus.
Wait time in Venus orbit, 450 days.

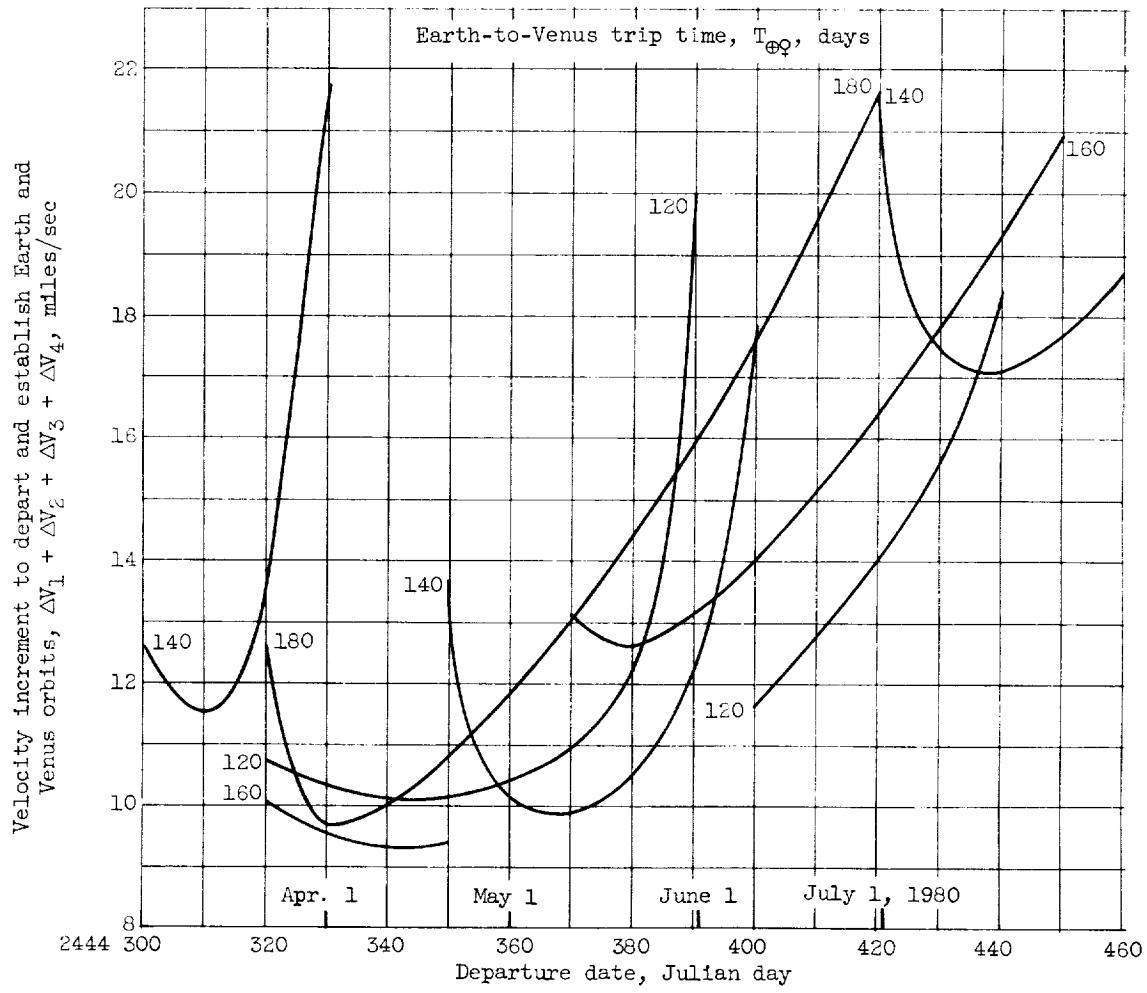


Figure 29. - Concluded. Velocity increments for 780-day round trip to Venus.
Wait time in Venus orbit, 450 days.

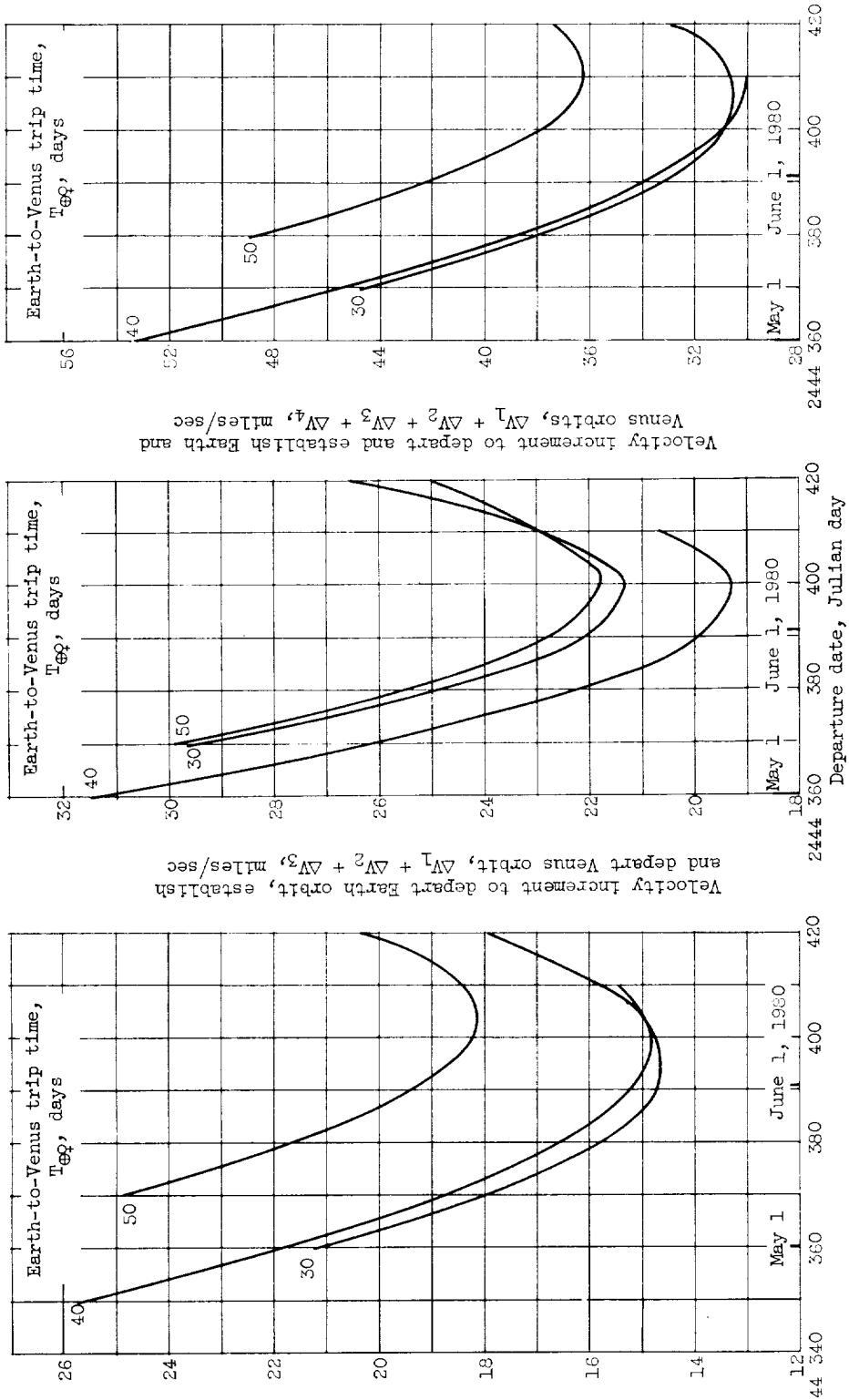
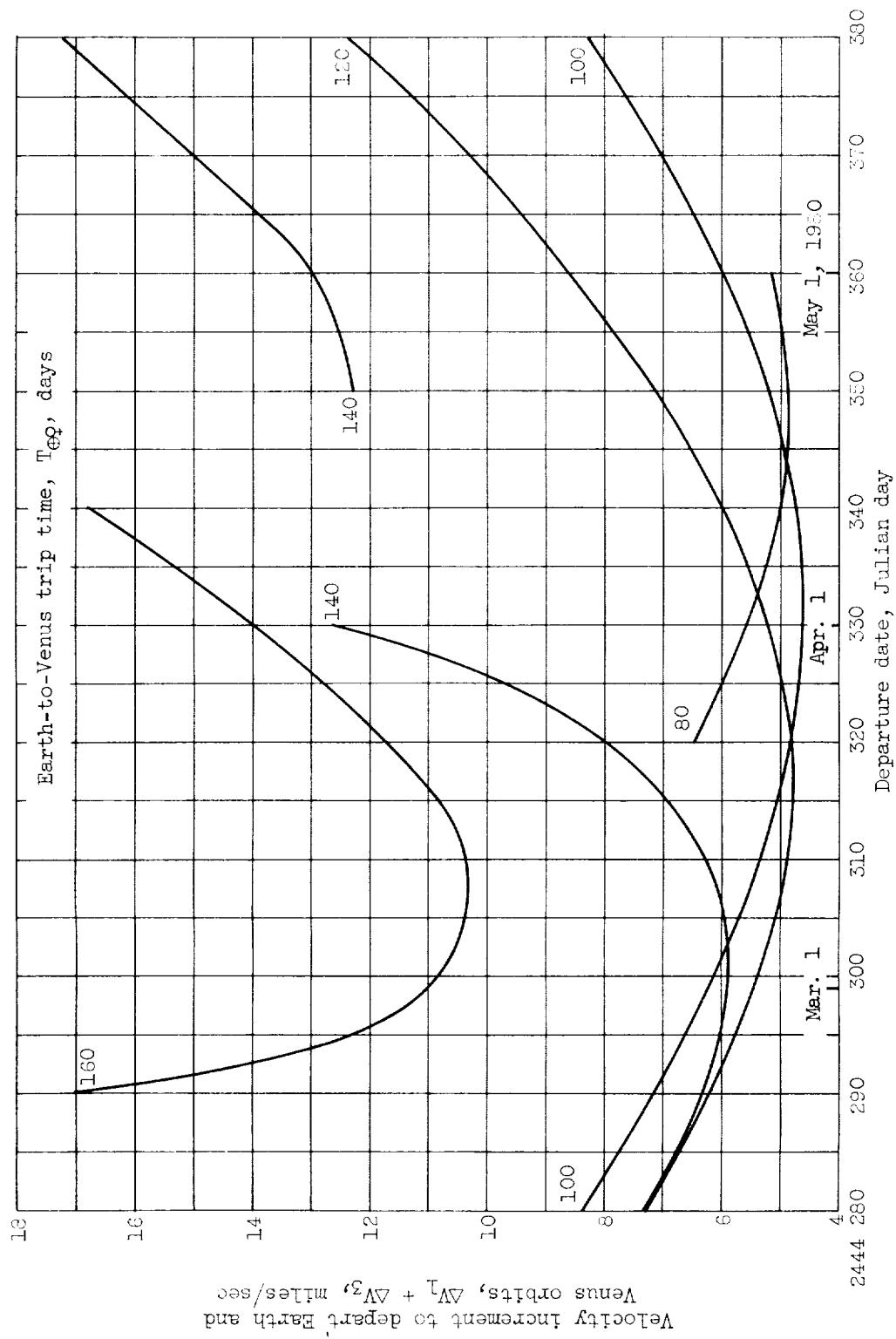


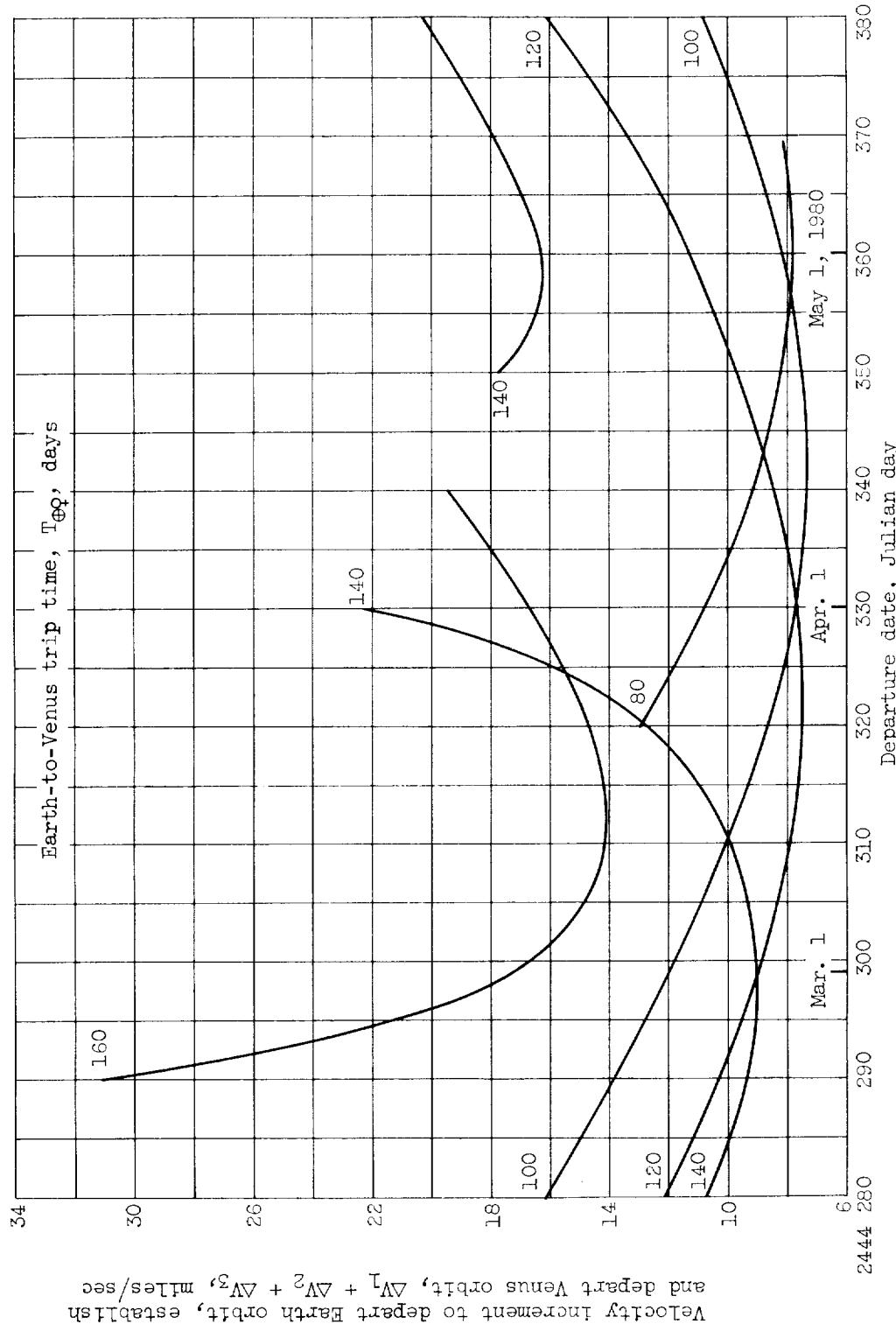
Figure 30. - Velocity increments for 580-day round trip to Venus. Wait time in Venus orbit, 510 days.

(a) Atmospheric braking at Venus and Earth.
 (b) Atmospheric braking at Earth.
 (c) All propulsive braking.



(a.) Atmospheric braking at Earth and Venus.

Figure 31. - Velocity increments for 700-day round trip to Venus. Wait time in Venus orbit, 510 days.



(b) Atmospheric braking at Earth.

Figure 31. - Continued. Velocity increments for 700-day round trip to Venus. Wait time in Venus orbit, 510 days.

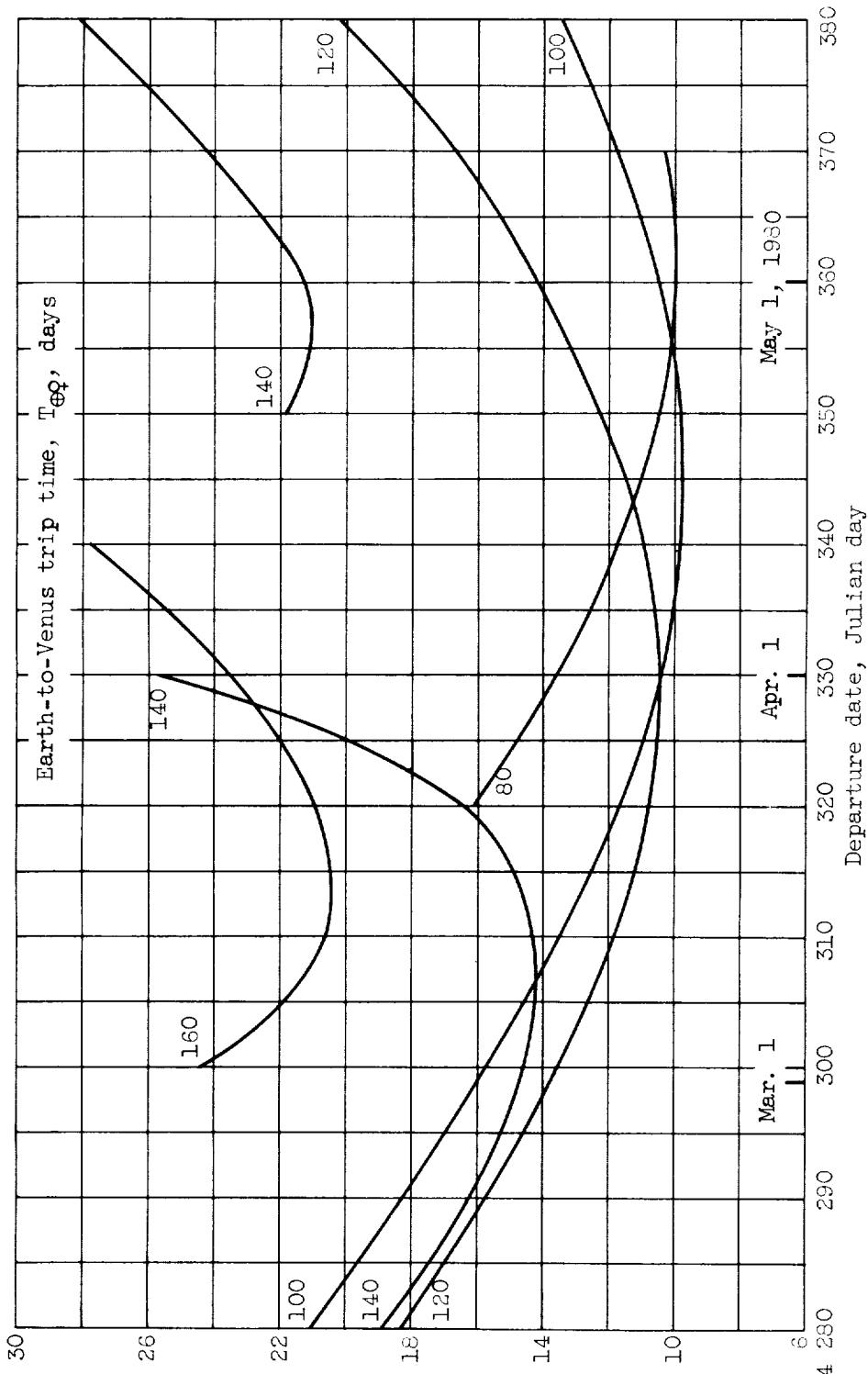
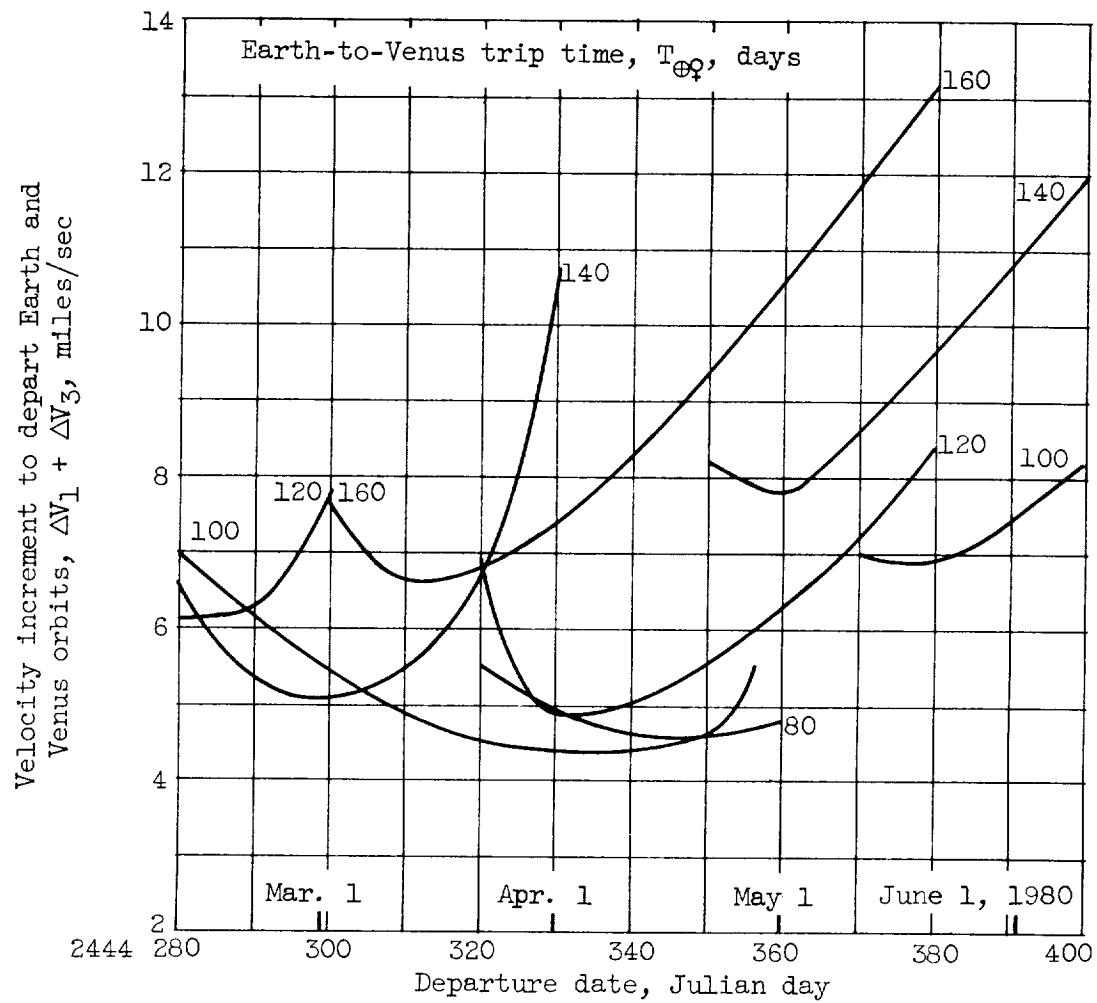


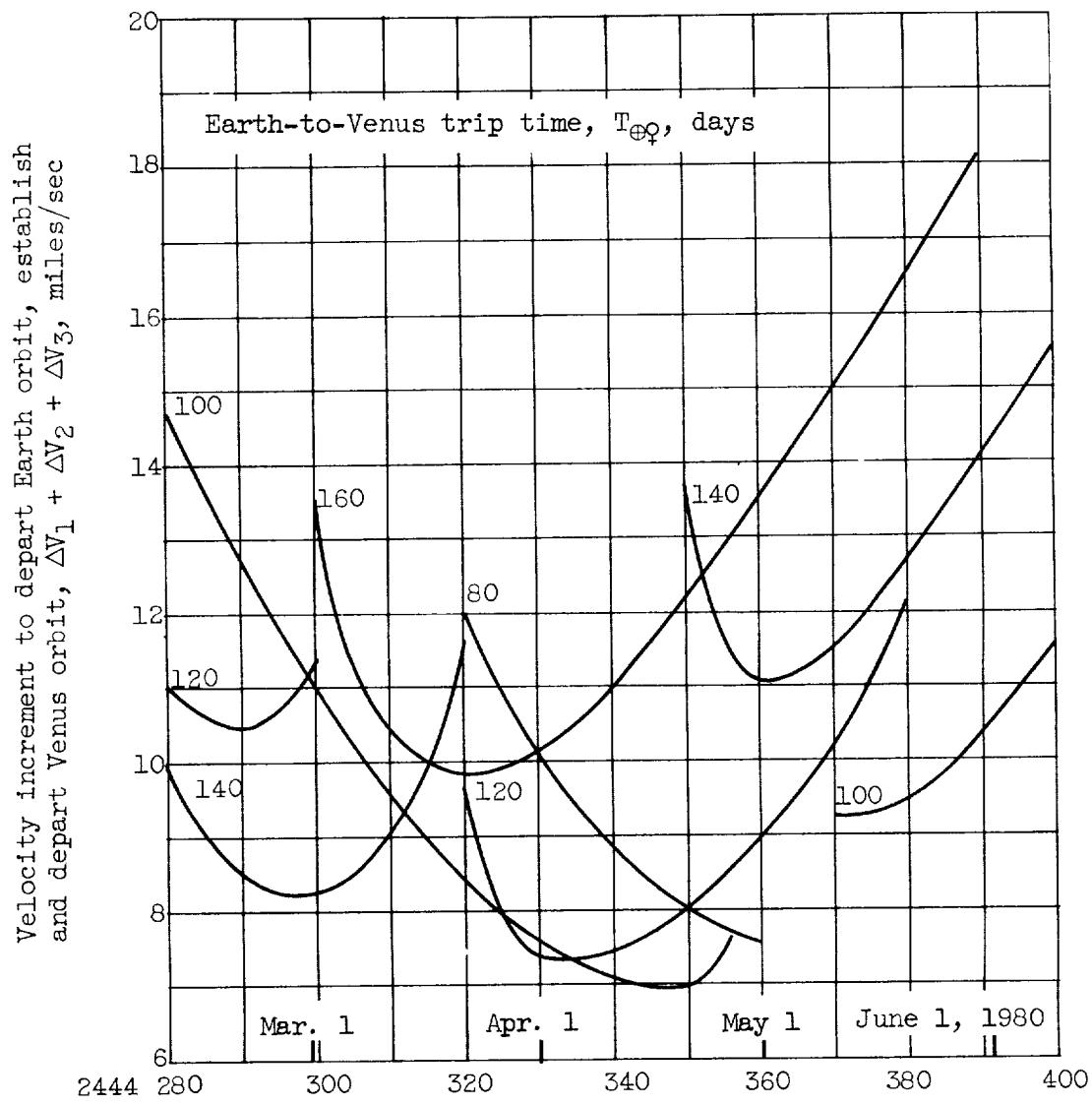
Figure 31. - Concluded. Velocity increments for 700-day round trip to Venus. Wait time in Venus orbit, 510 days.

(c) All propulsive braking.



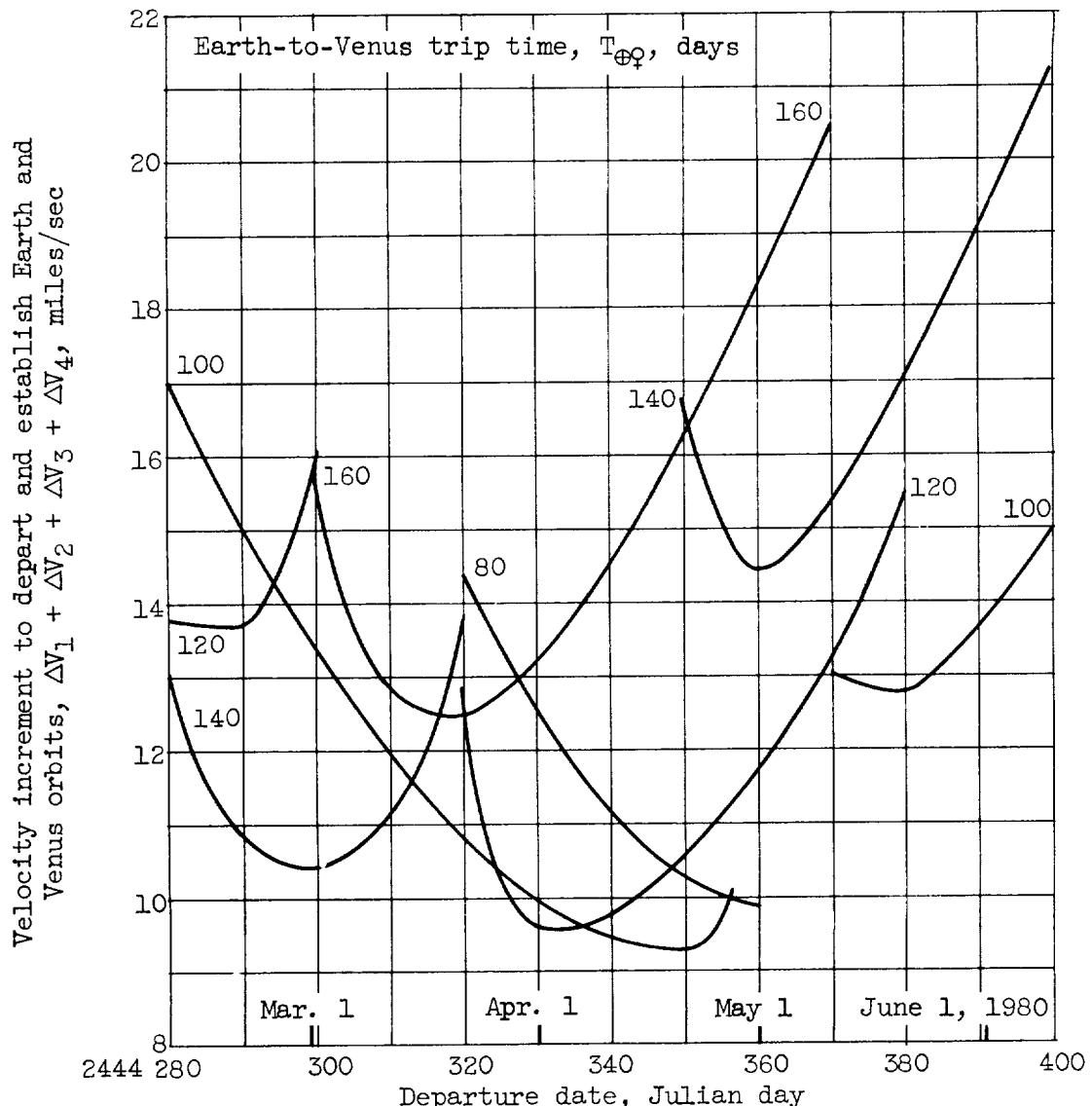
(a) Atmospheric braking at Venus and Earth.

Figure 32. - Velocity increments for 780-day round trip to Venus. Wait time in Venus orbit, 510 days.



(b) Atmospheric braking at Earth.

Figure 32. - Continued. Velocity increments for 780-day round trip to Venus. Wait time in Venus orbit, 510 days.



(c) All propulsive braking.

Figure 32. - Concluded. Velocity increments for 780-day round trip to Venus. Wait time in Venus orbit, 510 days.

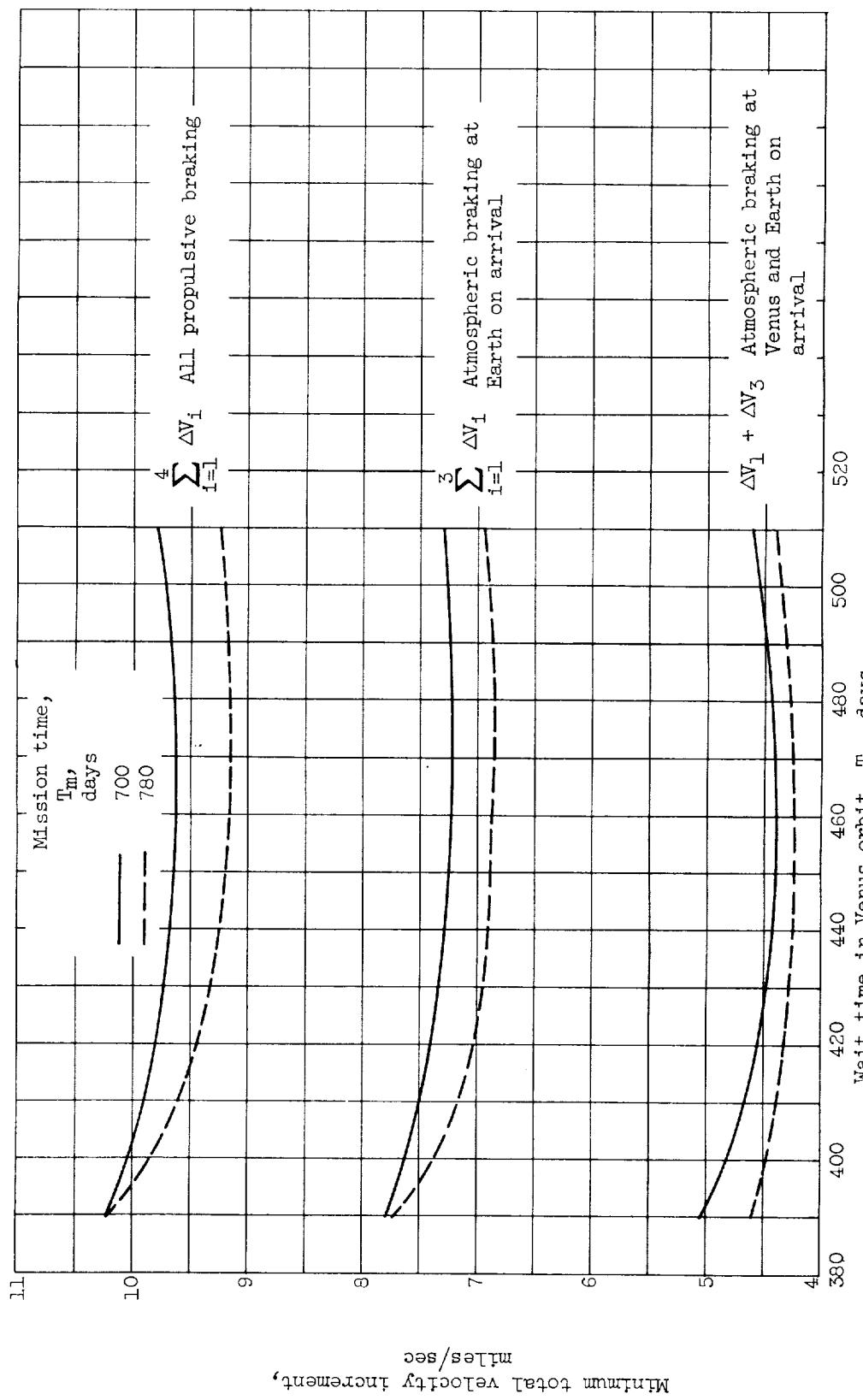
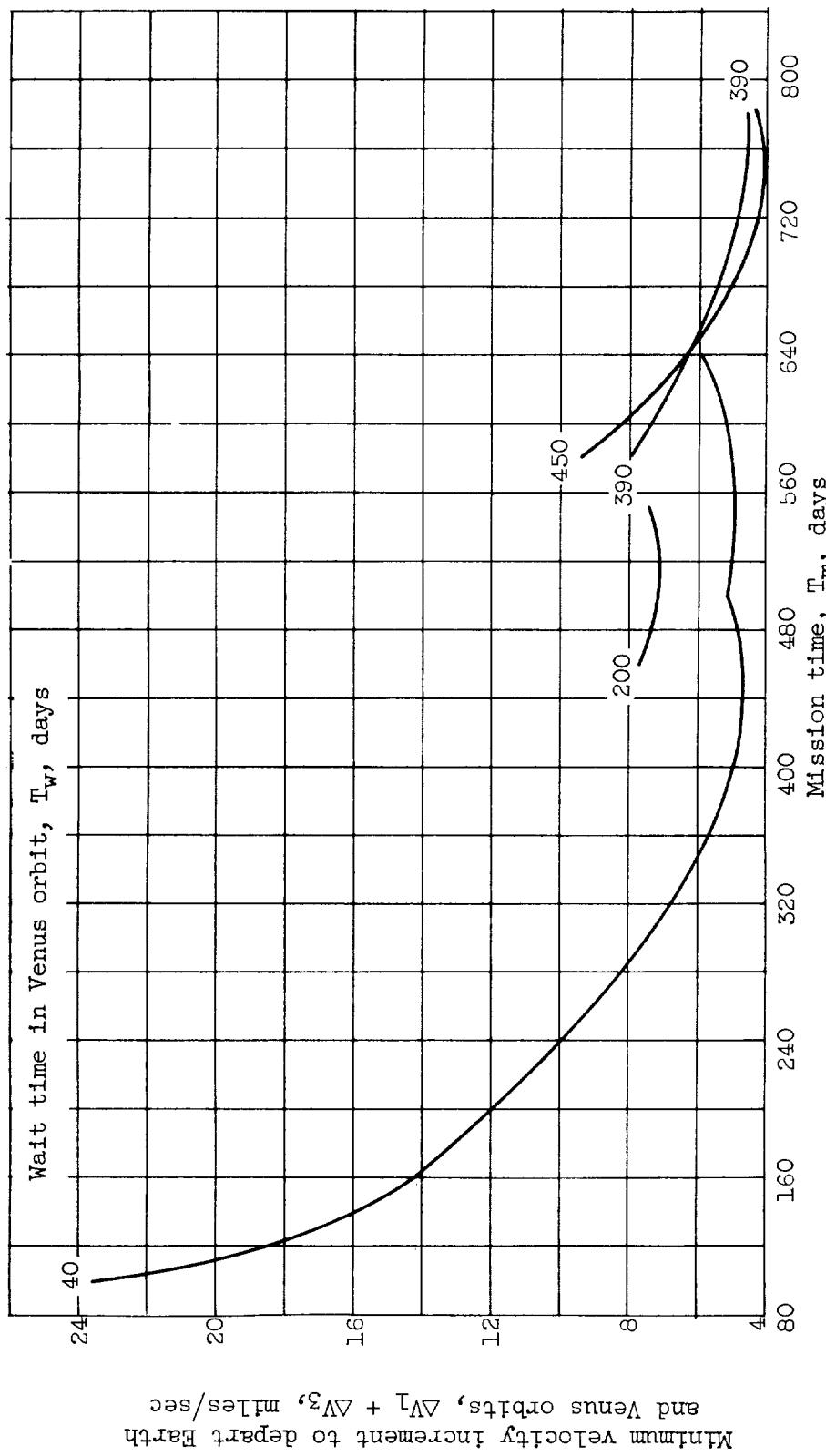
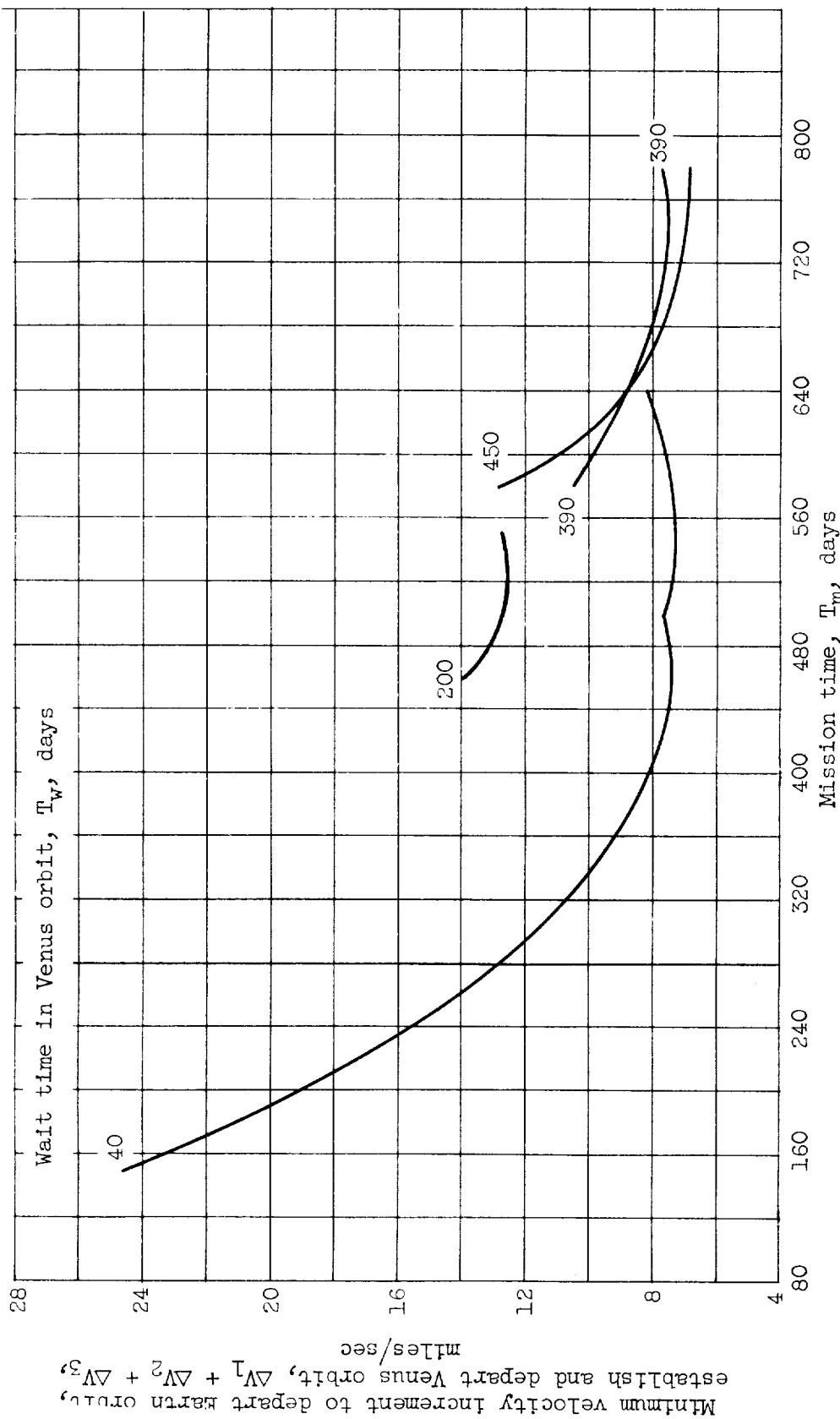


Figure 33. - Effect of wait time in Venus orbit on velocity-increment requirements for mission times of 700 and 780 days with atmospheric and all propulsive braking.



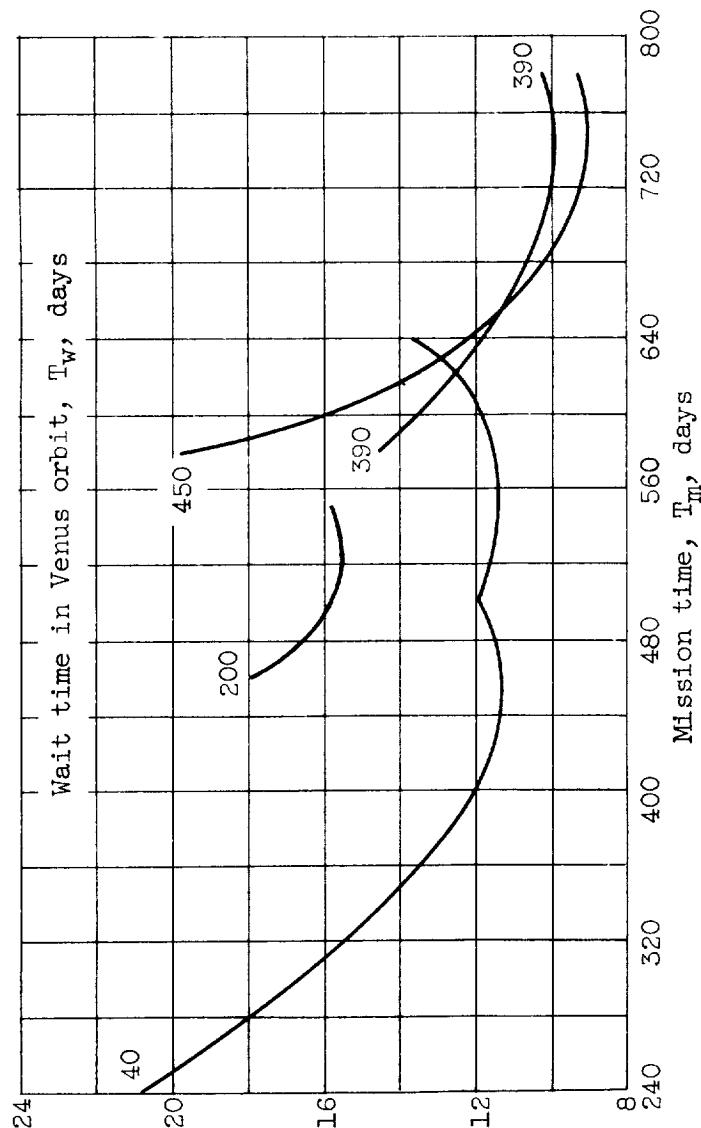
(a) Atmospheric braking at Venus and Earth.

Figure 34. - Effect of wait time in Venus orbit on total velocity increments required for round trips to Venus in 1980.



(b) Atmospheric braking at Earth.

Figure 34. - Continued. Effect of wait time in Venus orbit on total velocity increments required for round trips to Venus in 1980.



(c) All propulsive braking.

Figure 34. - Concluded. Effect of wait time in Venus orbit on total velocity increments required for round trips to Venus in 1980 and establish Earth and Venus orbits,

$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4, \text{ miles/sec}$$

Minimum velocity increment to depart

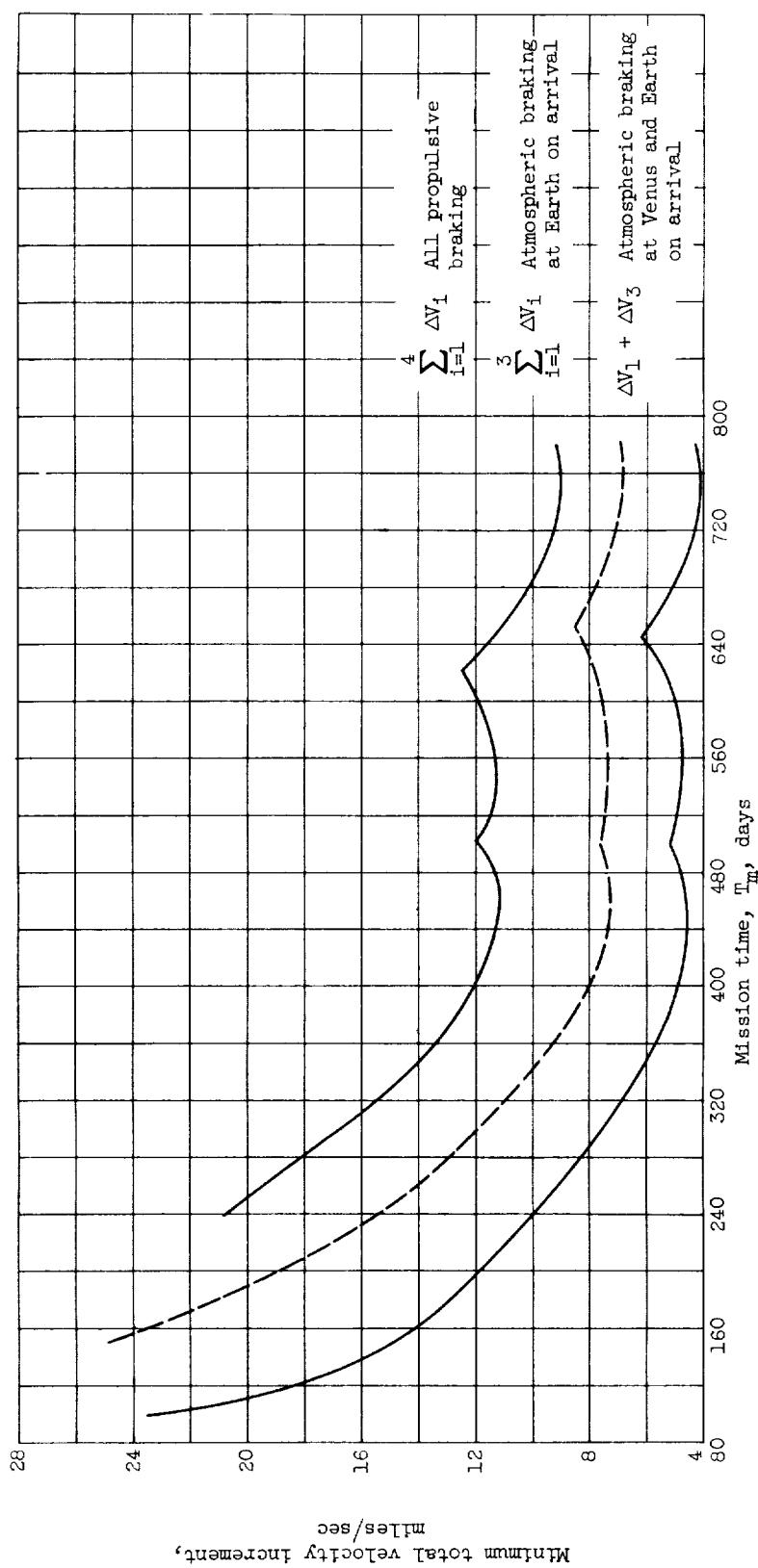
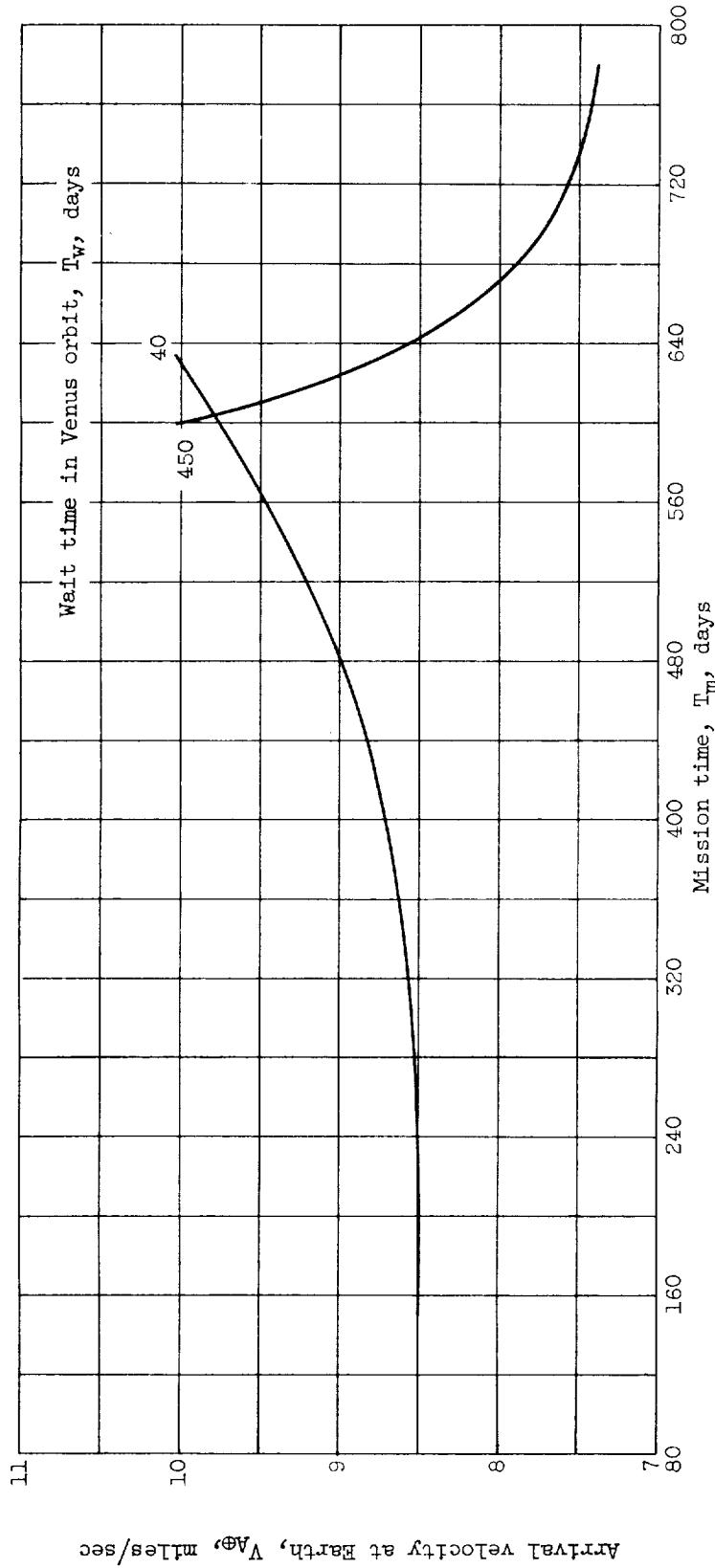


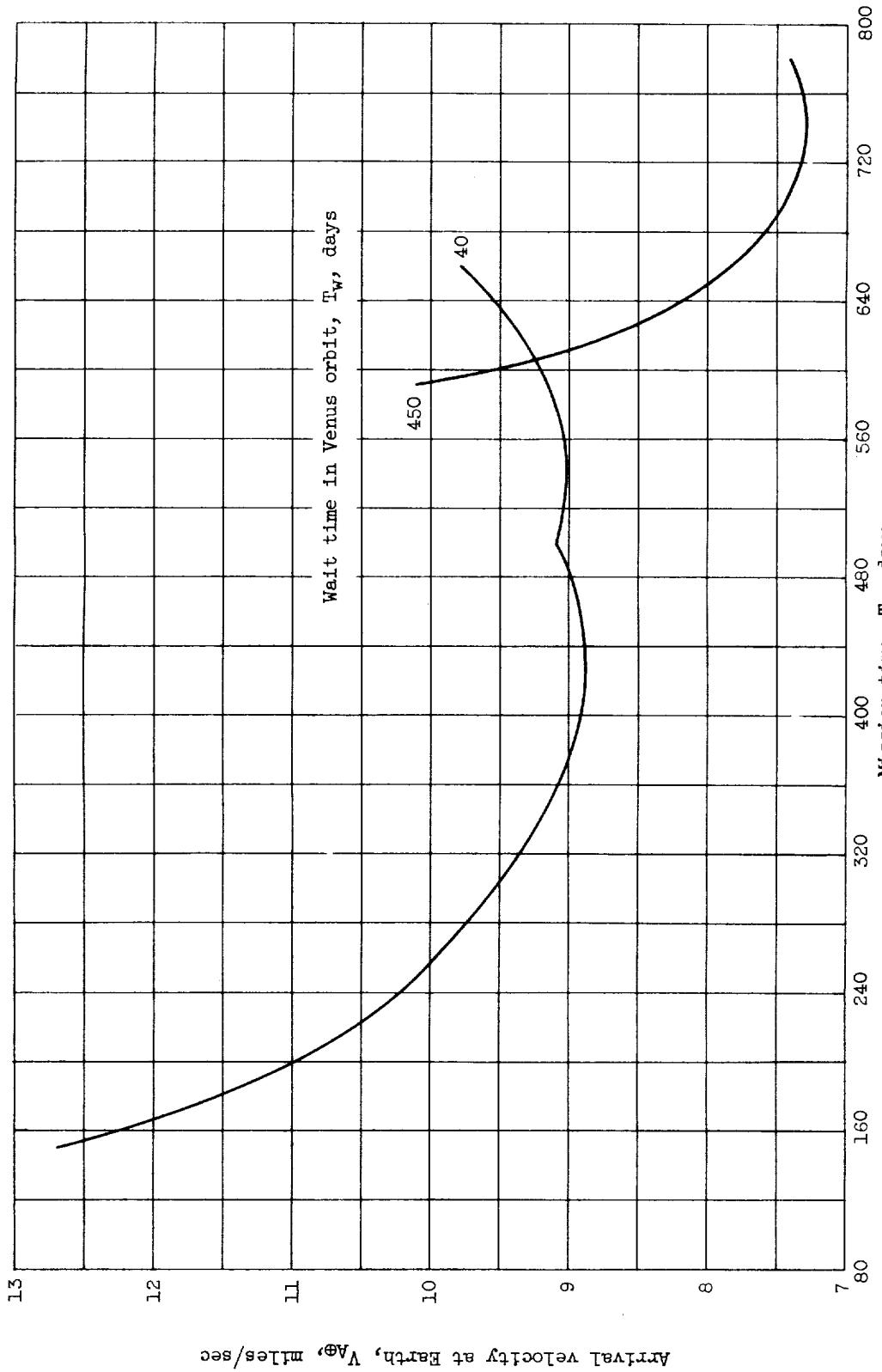
Figure 35. - Effect of atmospheric braking on total velocity increments required for round trips to Venus in 1980.



(a) Data for minimum velocity increment to depart Earth and Venus $\Delta V_1 + \Delta V_3$.

Figure 36. - Velocity on arrival at Earth from Venus as a function of round-trip mission time.

10 4



(b) Data for minimum total velocity increment $\sum_{i=1}^3 \Delta V_i$.

Figure 36. - Concluded. Velocity on arrival at Earth from Venus as a function of round-trip mission time.



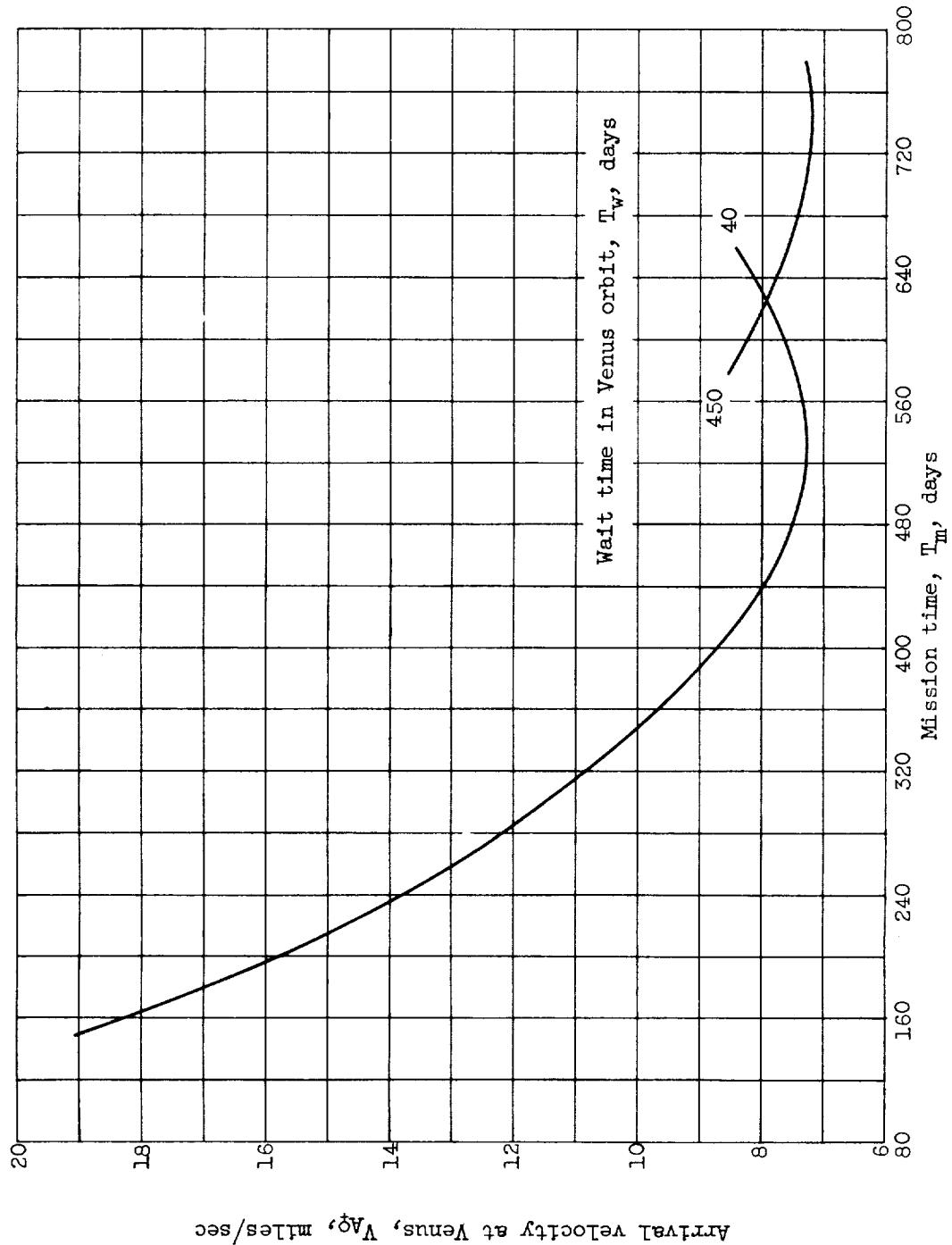


Figure 37. - Velocity on arrival at Venus from Earth as a function of round-trip mission time. Data for minimum velocity increment to depart Earth and Venus orbits $\Delta V_1 + \Delta V_3$.



